Information, Computation, Communication

Computer Architecture

Question

Now that we have created algorithms, how can we **construct systems that execute them**?

sum of first *n* integers

Input: n

Output: m

$$s \leftarrow 0$$

while $n > 0$
 $s \leftarrow s + n$
 $n \leftarrow n - 1$
 $m \leftarrow s$



From Algorithms to Computers

sum of first *n* integers

Input: n Output: m

 $s \leftarrow 0$ **while** n > 0 $s \leftarrow s + n$

 $n \leftarrow n - 1$

 $m \leftarrow s$



sum of first *n* integers

Input: r1 Output: r2

1: copy r3, 0 2: jump_egz r1, 6

3: add r3, r3, r1

4: add r1, r1, -1

5: jump 2

6: copy r2, r3

sum of first *n* integers

Input: r1 Output: r2

1: 0100010100000000

2: 0101100000001010 3: 0001001001011010

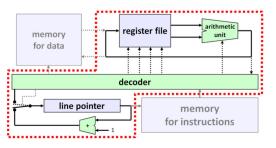
4: 0001010101000000

5: 0000101010101111

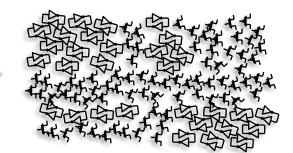
6: 0101000001100101

Software

Hardware



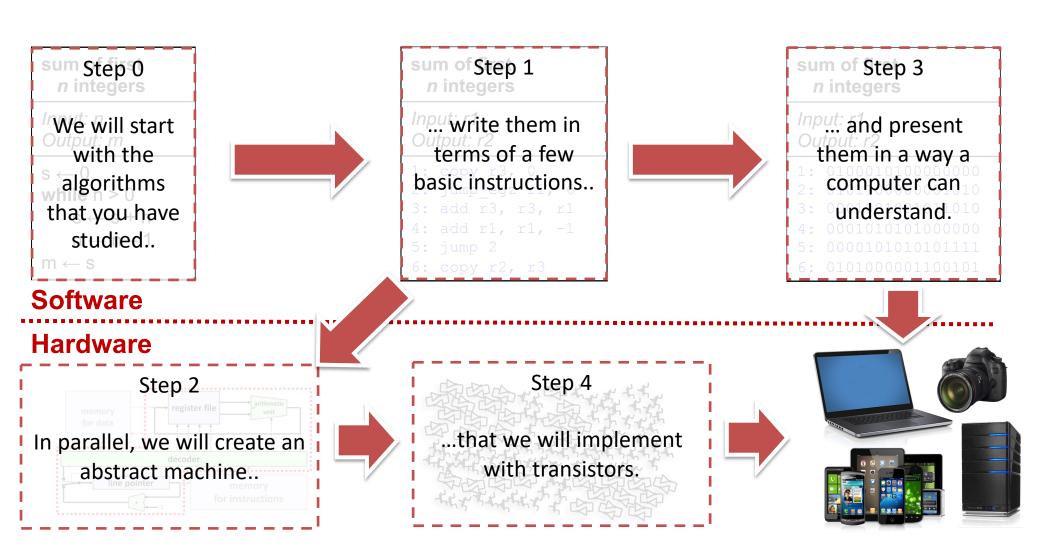








From Algorithms to Computers



From Algorithms to Computers (Step 0)

- In order to describe the idea of an algorithm, we use pseudo code.
- Let's consider an example

sum
Input: n
Output: m
s ← 0
while n > 0
s ← s + n
n ← n − 1
$m \leftarrow s$

From Algorithms to Computers (Step 1)

sum of first *n* integers

Input: n Output: m

$$s \leftarrow 0$$

while $n > 0$
 $s \leftarrow s + n$
 $n \leftarrow n - 1$
 $m \leftarrow s$



sum of first *n* integers

Input: r1 Output: r2

```
1: copy r3, 0
2: jump_egz r1, 6
3: add r3, r3, r1
4: add r1, r1, -1
5: jump 2
6: copy r2, r3
```

Software

Hardware

We will rewrite algorithm in terms of a few basic instructions, which will have physical counter parts (hardware to perform them).



Rewrite Algorithm using Basic Instructions

Our machine needs to be able to remember values, e.g., value for s

sum of first n integers

Input: n Output: m

$$s \leftarrow 0$$

while n > 0

$$n \leftarrow n - 1$$

$$m \leftarrow s$$

In all computers, values are store in so-called registers, which are physical implementations of variables with a fix number of bits (usually 32 or 64 bits)

Registers

- ➤ A **register** is the physical implementation of the notion of variable
- ➤ A machine usually has a small number of registers (a few dozen)
- For large data (array, list,..) we will use external memory (RAM)
- Registers are usually represented by r1, r2, r3,...
- We replace all arbitrary variable names with register names
 - n → r1
 - m → r2
 - •s → r3

Step 1.1

sum

Input: $n \rightarrow r1$ Output: m → r2

$$s \leftarrow 0 \rightarrow r3$$



while n > 0

$$s \leftarrow s + n$$

$$n \leftarrow n - 1$$

$$m \leftarrow s$$



sum

Input: r1

Output: r2

while r1 > 0

$$r3 \leftarrow r3 + r1$$

$$r1 \leftarrow r1 - 1$$

$$r2 \leftarrow r3$$

Next...

sum

Input: r1

Output: r2

while r1 > 0

$$r3 \leftarrow r3 + r1$$

We will need to assign values to registers

We will use a basic instruction, e.g., "copy r3, 0" for r3 ← 0 or "copy r2, r3" for r2 ← r3

Step 1.2

sum

Input: **r1**

Output: r2

$$r3 \leftarrow 0$$
while $r1 > 0$

$$r3 \leftarrow r3 + r1$$

$$r1 \leftarrow r1 - 1$$

$$r2 \leftarrow r3$$



sum

Input: r1

Output: r2

copy r3, 0 while r1 > 0 $r3 \leftarrow r3 + r1$ $r1 \leftarrow r1 - 1$ copy r2, r3

Next...

sum

Input: **r1**Output: **r2**

copy r3, 0
while r1 > 0 $r3 \leftarrow r3 + r1$ $r1 \leftarrow r1 - 1$ copy r2, r3

We will need to
assign new values
to registers
after applying
arithmetic operations

We will use basic instructions, e.g., "add r3, r3, r1" for r3 ← r3 + r1

Step 1.3

sum of first *n* integers

Input: r1

Output: r2

copy r3, 0
while r1 > 0
 r3 \leftarrow r3 + r1
 r1 \leftarrow r1 - 1
copy r2, r3



sum of first n integers

Input: r1

Output: r2

copy r3, 0
while r1 > 0
 add r3, r3, r1
 add r1, r1, -1
copy r2, r3

Basic Instructions

- ➤ There is a limited number of instructions, e.g.,
 - copy for assignment
 - add for addition
 - mul for multiplication
- All instructions (i) have a single result, (ii) take one or two registers (or constants) as operands
- Instructions are written in the following form:
 name destination, operand1, operand2
- Every computation in an algorithm is rewritten in these basic instructions, e.g., a ← a * (b + c) with a in r1, b in r2 and c in r3 could be rewritten as. add r2, r2, r3 mul r1, r1, r2

Next...

Input: r1 Output: r2 copy r3, 0 while r1 > 0 add r3, r3, r1

add r1, r1, -1

copy r2, r3

- ➤ All control structures (if-conditions, while-loop, for-loops, ..) will be replaced by **jumps to labels**
- We will use line numbers as labels
- We will have a few different (conditional) jump instructions, e.g.,
 - jump 2: always jump to line 2
 - •jump_egz r1, 6:
 jump to line 6, if r1 is
 equal to zero
 - jump_eg r1, r2, 6: jump to line 6, if r1 is equal to r2

Step 1.4

sum

Input: **r1**Output: **r2**

copy r3, 0
while r1 > 0
 add r3, r3, r1
 add r1, r1, -1

copy r2, r3

sum

Input: **r1**Output: **r2**

1: copy r3, 0

2: jump egz r1, 6

3: add r3, r3, r1

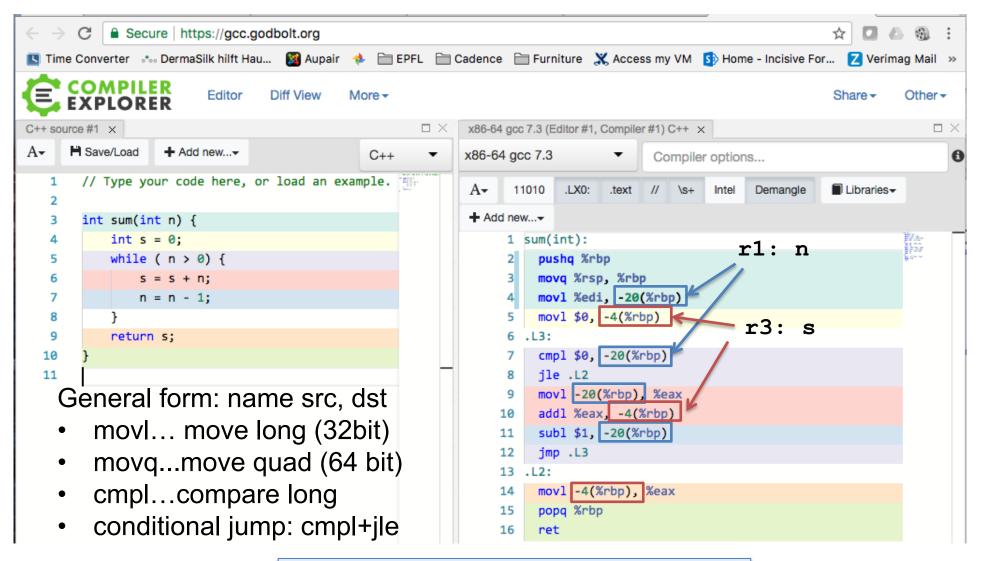
4: add r1, r1, -1

5: jump 2

6: copy r2, r3

This is a program in assembly (or assembler) language.

Example in x86 Assembly Language (Intel)



Example in x86 Assembly Language (Intel)

Example in C

```
#include <stdio.h>
int main () {
  int a = 10:
  int b = 45;
  int add, mul;
    asm ( "addl %ebx, %eax;" : "=a" (add) :
   asm ( "imull %%ebx, %%eax;" : "=a" (mul) :
  printf("Add = %d \n", add);
  printf("Mul = %d \n", mul);
Compile with: g++ assembly.c -o assembly -g
Run: ./assembly
Add = 55
Mul = 450
```



```
## %bb.0:
       pushq
              %rbp
       .cfi_def_cfa_offset 16
       .cfi_offset %rbp, -16
              %rsp, %rbp
       movq
       .cfi_def_cfa_register %rbp
       pushq
              %rbx
              $24, %rsp
       subg
       .cfi_offset %rbx, -24
       movl $10, -12(%rbp)
       movl $45, -16(%rbp)
       movl -12(%rbp), %eax
       movl
              -16(%rbp), %ebx
       ## InlineAsm Start
       addl
              %ebx, %eax
       ## InlineAsm End
       movl
              %eax, -20(%rbp)
              -12(%rbp), %eax
       movl
       movl
              -16(%rbp), %ebx
       ## InlineAsm Start
       imull
              %ebx, %eax
```

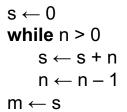
Summary

- We use "registers" to mimic variables in hardware
- ➤ We rewrite our program in terms of basic "instructions"
 - instructions to load/copy values into a registers
 - instructions for arithmetic (and binary) operations
 - instructions to jump to another instruction under some condition
- We use a restricted set of previously defined instructions
- ➤ E.g., ARM or Intel processors have their own set of instructions (x86, ARM, Risc-V ISA instruction set architecture)

From Algorithms to Computers (Step 2)

sum of first *n* integers

Input: n Output: m





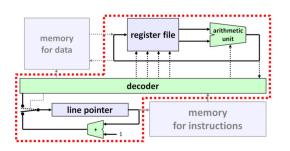
sum of first *n* integers

Input: r1 Output: r2

1: copy r3, 0 2: jump_egz r1, 6 3: add r3, r3, r1 4: add r1, r1, -1 5: jump 2 6: copy r2, r3

Software

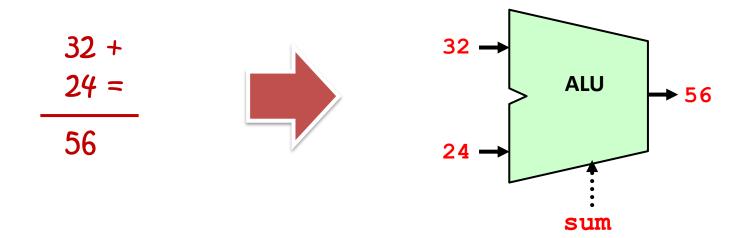
Hardware





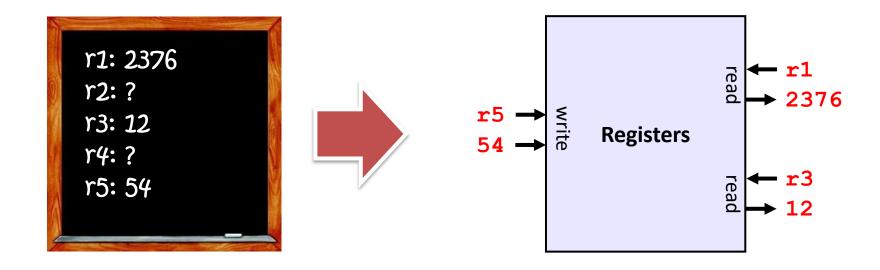
What do we need to calculate?

➤ Arithmetic logic unit (ALU) for arithmetic (and bitwise) operations (+,-,*,/,mod, &, |, ^,<<,>>,..)

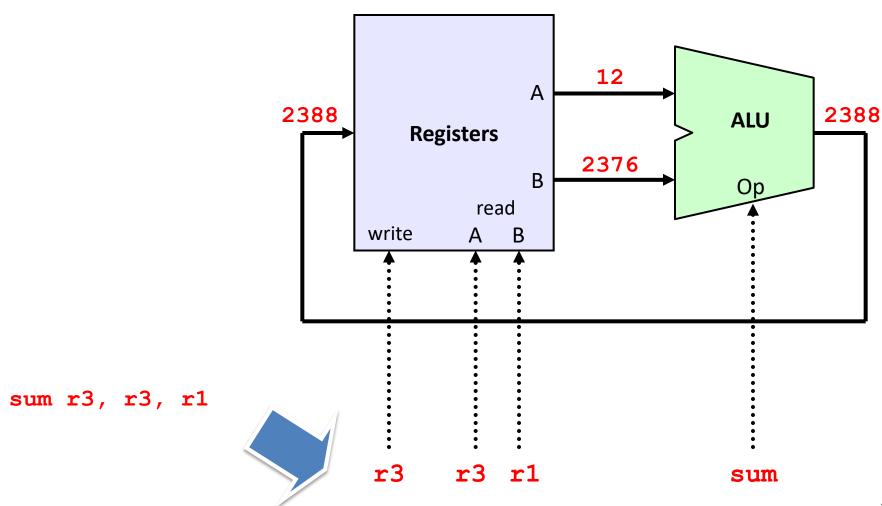


What do we need to calculate?

Registers to save the values of the operands and the result

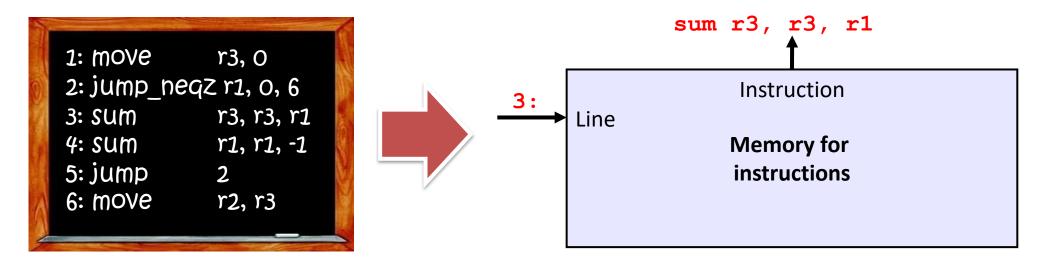


A Circuit to Calculate

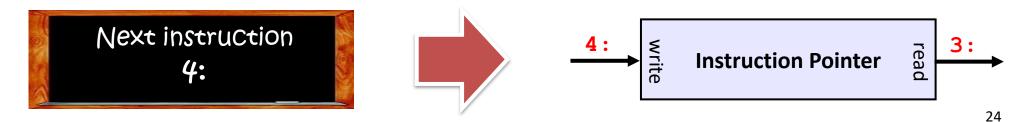


What else do we need?

Our algorithm or program needs to be stored somewhere

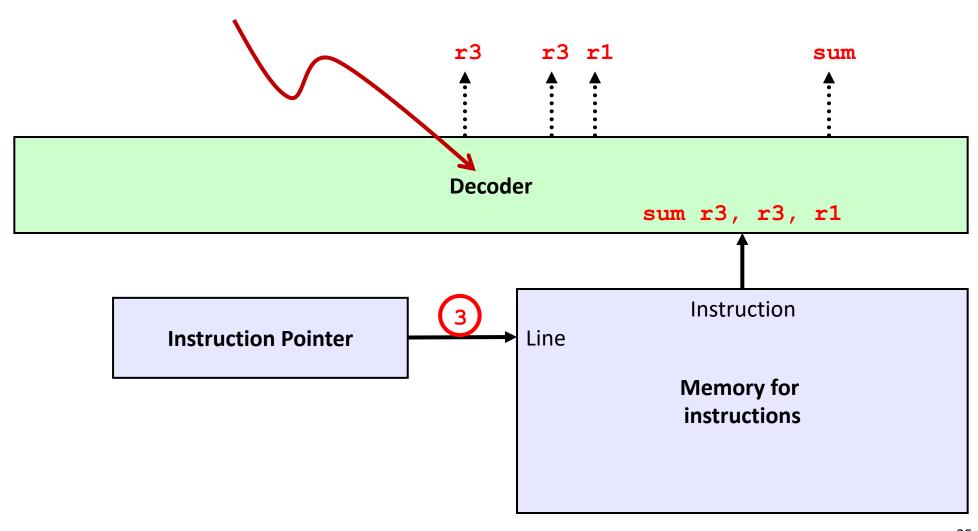


We need a way to control where we are



How to control where we are?

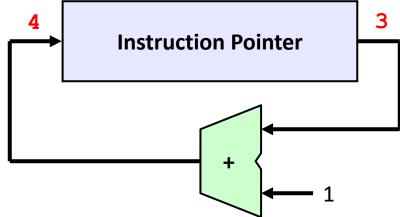
A simple circuit that separates the elements in an instruction



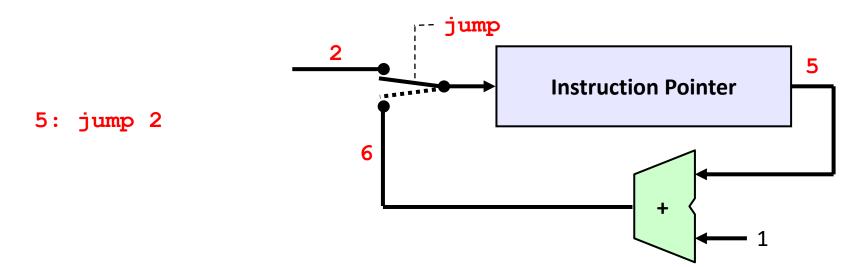
How to control where we are?

Usually we pass to the next line

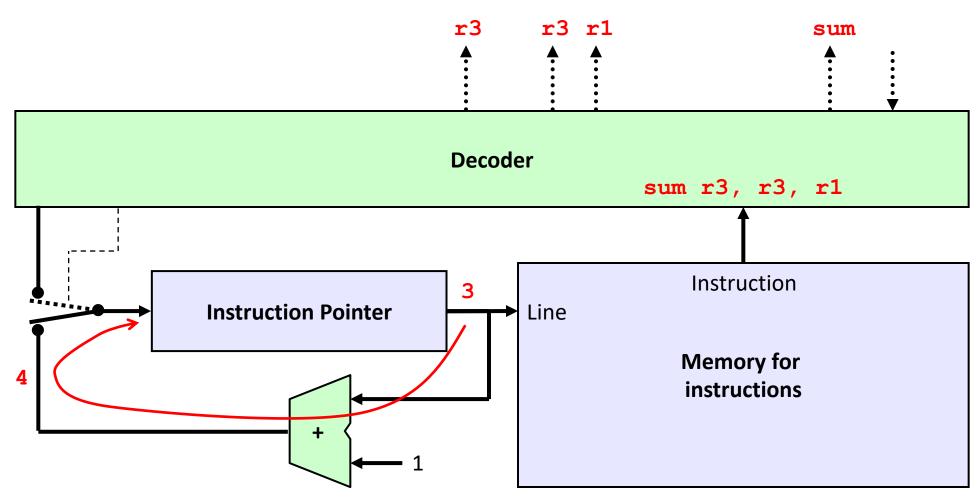
3: sum r3, r3, r1



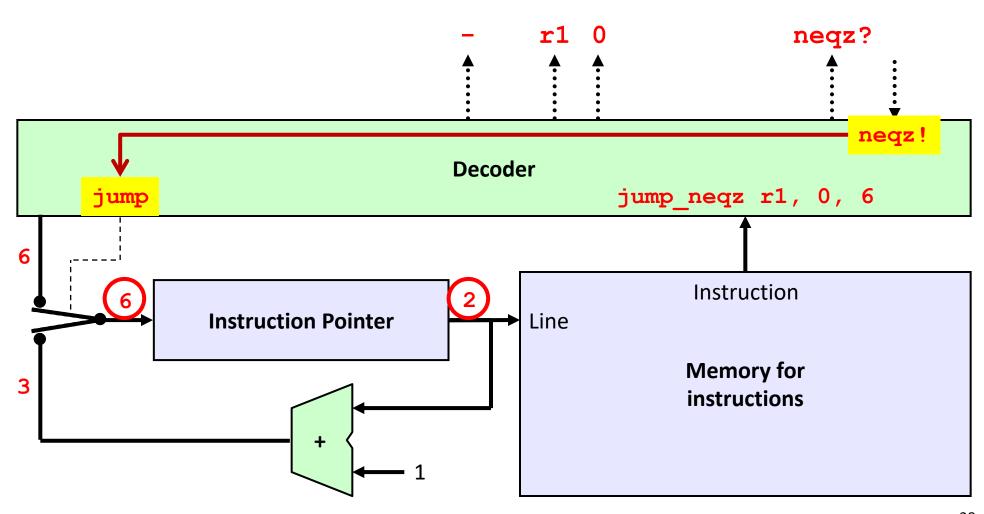
▶ If we get an instruction jump, we switch to the given line



A Circuit to Control the Instruction Pointer

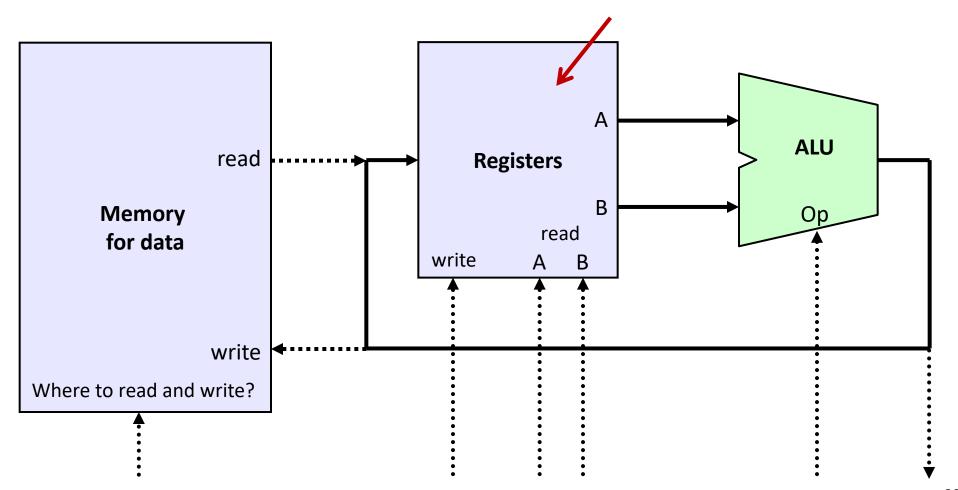


A Circuit to Control the Instruction Pointer

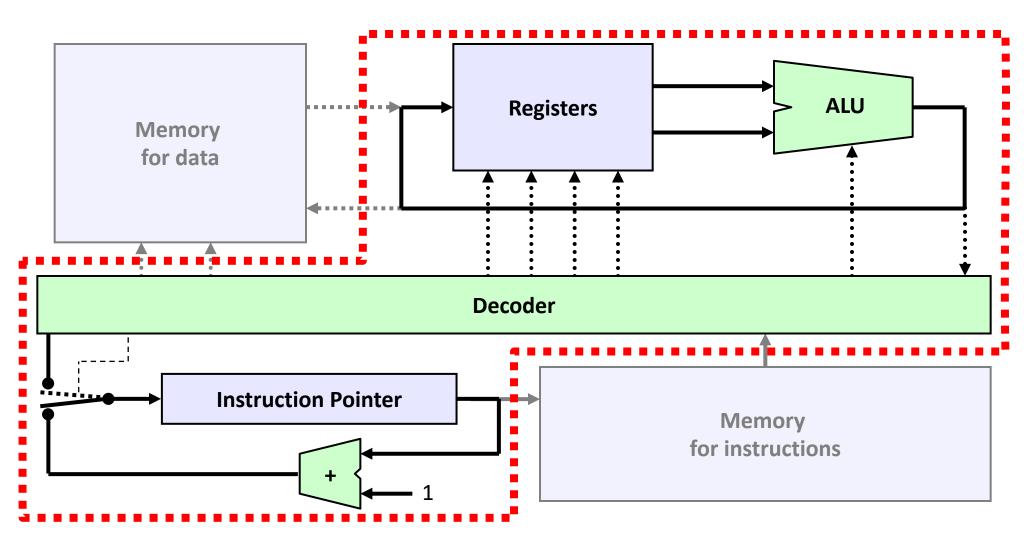


Memory to Store More Data

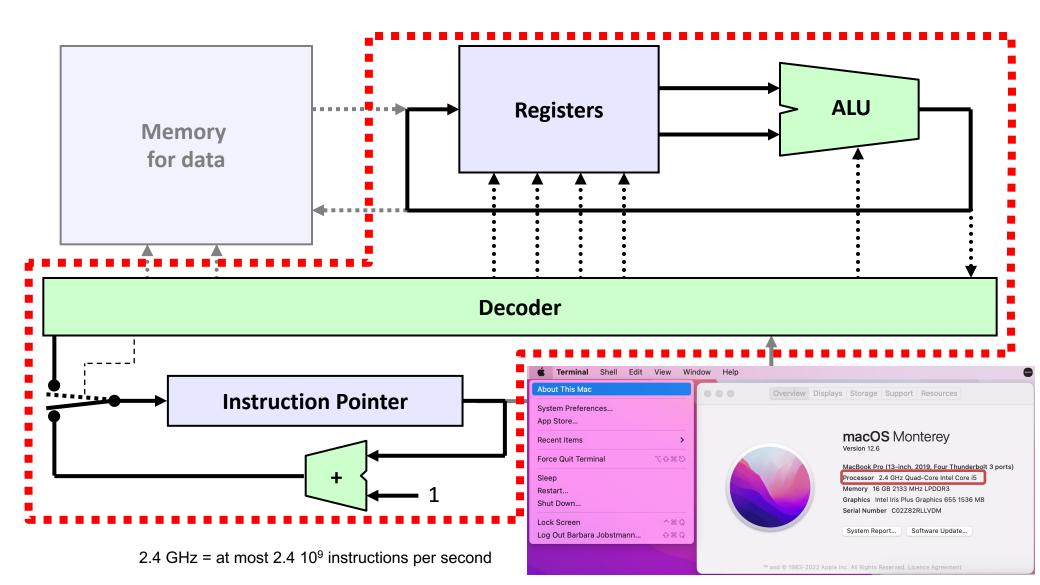
Relatively small: only a few dozens registers



Processor or Central Processing Unit (CPU)

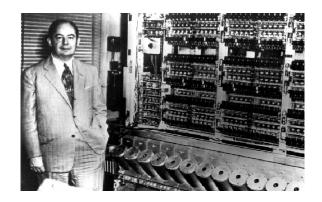


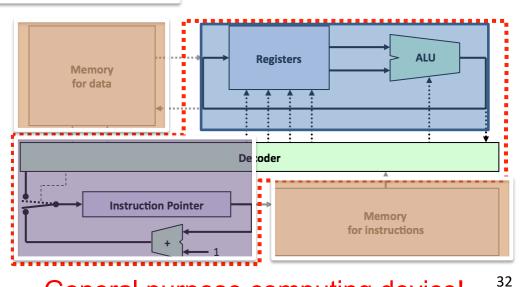
Processor or Central Processing Unit (CPU)



von Neumann Architecture

- Architecture based on a 1945 description by John von Neumann
 - Processing unit that contains an arithmetic logic unit and processor registers
 - Control unit that contains an instruction register and program counter
 - Memory that stores data and instructions
 - External mass storage
 - Input and output mechanisms

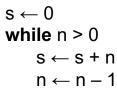




From Algorithms to Computers (Step 3)

sum of first *n* integers

Input: n Output: m







sum of first *n* integers

Input: r1 Output: r2

1: copy r3, 0 2: jump_egz r1, 6 3: add r3, r3, r1 4: add r1, r1, -1

5: jump 2

6: copy r2, r3

sum of first *n* integers

Input: r1 Output: r2

1: 0100010100000000 2: 0101100000001010

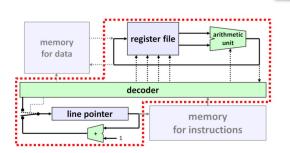
3: 0001001001011010 4: 0001010101000000

5: 00001010101000000

6: 0101000001100101

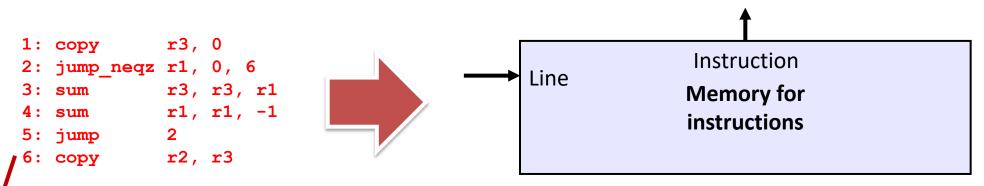
Software

Hardware

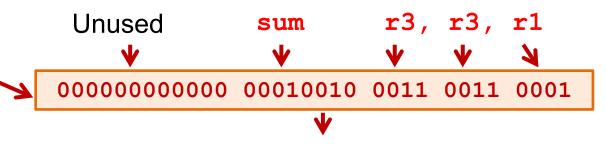




How to Encode Instructions?



- We can invent a simple encoding (cf. Lesson on "Information Representation"):
 - A few bits for the name of the instruction (with 8 bits we can encode 256 different instructions)
 - A few bits for the registers (with 4 bits we can address 16 registers, so with 12 bits we can store two operands and one destination register)
 - And so on and so forth...
- So we can get by with 32 or 64 bits, as if to encode a typical integer



How to Encode Instructions?

Assembly Language

sum

Input: **r**1

Output: r2

1: copy r3, 0

2: jump negz r1, 6

3: add r3, r3, r1

4: add r1, r1, -1

5: jump 2

6: copy r2, r3

Machine Language in Binary

sum

Input: r1

Output: r2

1: 0000000100110000000

2: 00000101000101100000

3: 00000010001100110001

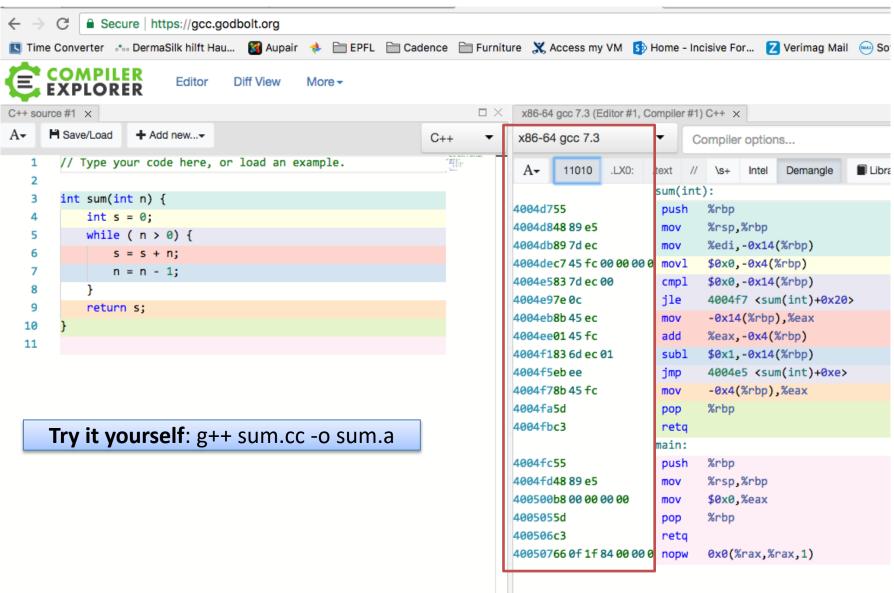
4: 00000010000100011111

5: 0000010000100000000

6: 00000001001000110000



Example in Machine Language (Intel)



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From Algorithms to Computers

sum of first *n* integers

Input: n Output: m

 $s \leftarrow 0$ while n > 0 $s \leftarrow s + n$ $n \leftarrow n - 1$

 $m \leftarrow s$



sum of first *n* integers

Input: r1
Output: r2

1: copy r3, 0 2: jump_egz r1, 6 3: add r3, r3, r1

4: add r1, r1, -1

5: jump 2

6: copy r2, r3

sum of first *n* integers

Input: r1 Output: r2

1: 0100010100000000

2: 010110000001010

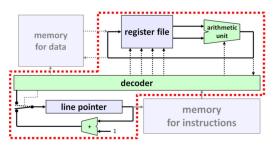
3: 0001001001011010 4: 0001010101000000

5: 00001010101000000 5: 0000101010101111

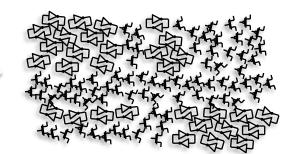
6: 0101000001100101

Software

Hardware





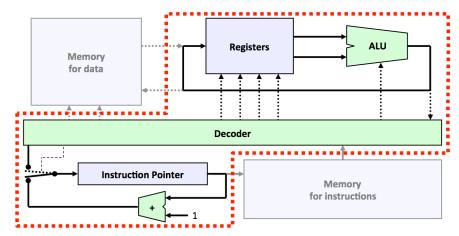




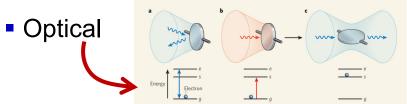


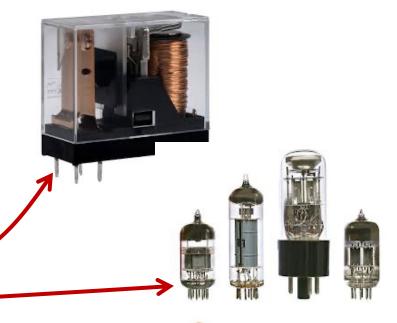
Technologies

- So far, our machine is abstract, completely independent of the underlying technology
- Even binary encoding is not a necessity



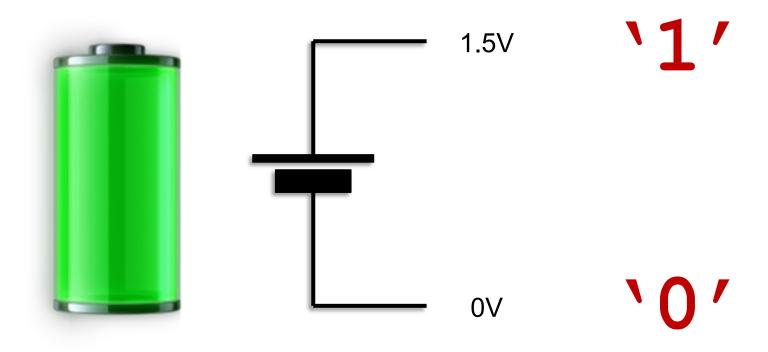
- Different technologies are possible:
 - Electro-mechanical (e.g., relays)
 - Electronic (e.g., tubes or transistors)







A Battery has Two Voltage Levels



Very well suited to represent information in binary

Switch (Interruptible Wire)



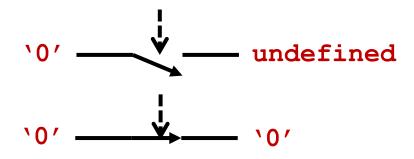
Does not propagate its state if the connection is open

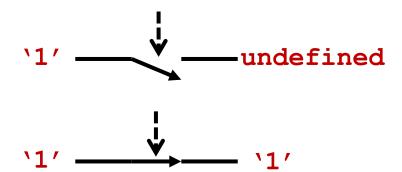




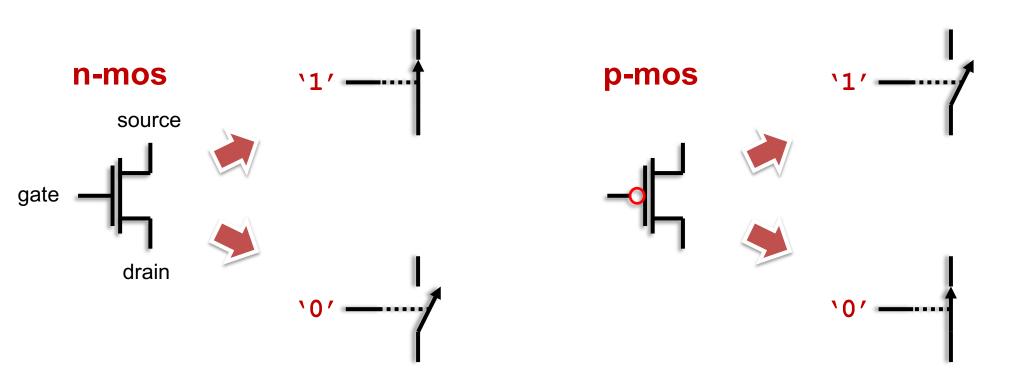
Propagates its states if the connection is closed

Transistor = Controllable Switch

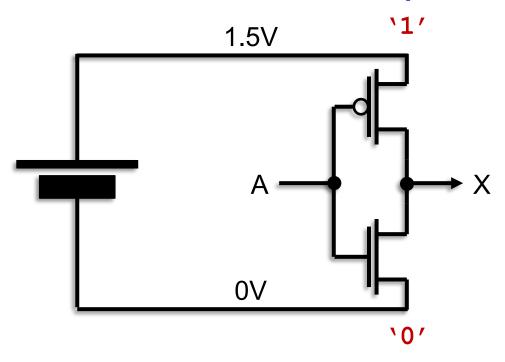




Transistor = Controllable Switch

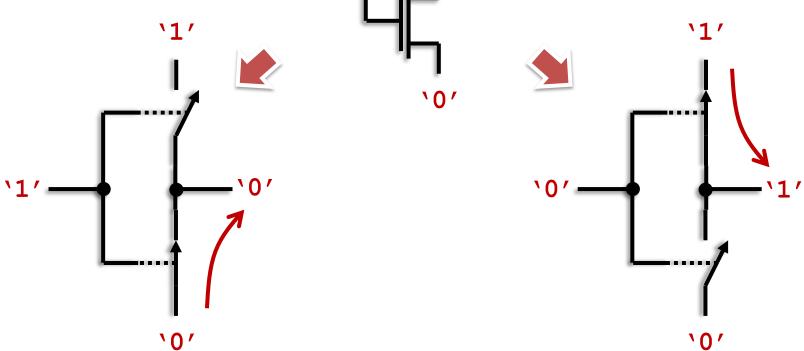


Inverter (NOT Gate)



Gate = electronic circuit that can do basic computation

Inverter (NOT Gate) Truth table X **Logic Symbol** 0



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Truth Table of a Circuit

- Describes the function of a circuit
- ► For instance, consider a circuit with 2 inputs and 1 output
 - 2² = 4 combinations (possible states)

Input 1	Input 2	Output
0	0	,
0	1	Ş
1	0	,
1	1	?

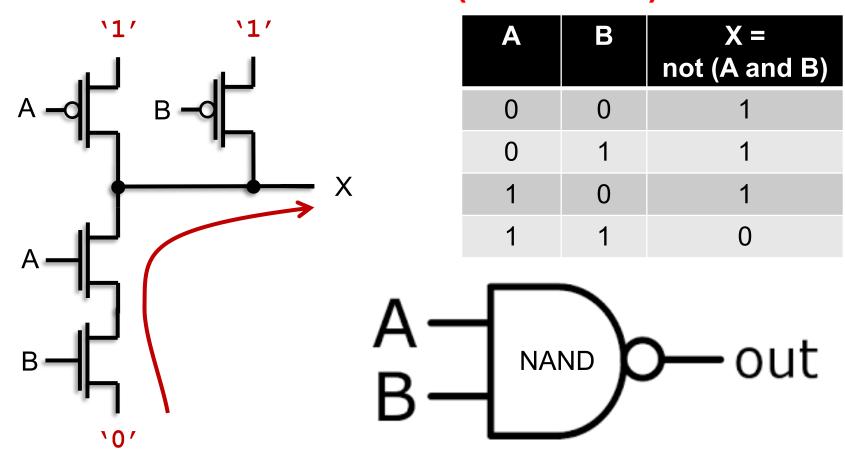
► Inverter from previous slide (1 input, 1 output)

Input	Output
0	1
1	0

Drawing a Truth Table

Input 1	Input 2	Input 3	Output
0	0	0	0
0	0	1	1
0	1	0	•••
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

A Circuit "NAND" (NOT AND)

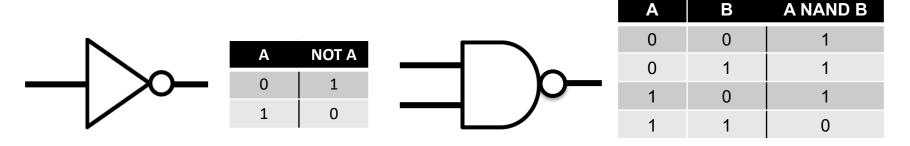


The only way to get a '0' is to put two '1' at the inputs A NAND B: the output is '0' only if A AND B are '1'

Circuits for Other Functions

We already know NOT and NAND

Boolean function: $\mathbb{B}^n \to \mathbb{B}^m$



NOT and NAND gates can implement any Boolean function. How many different Boolean functions with two inputs exist?

•	Α	В	0	AND	F2	Α	F4	В	XOR	OR on
ı	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	1	1	1	1
	1	0	0	0	1	1	0	0	1	1
	1	1	0	1	0	1	0	1	0	1

Α	В	NOR	F9	NOT B	F11	NOT A	F13	NAND	1
0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1

Do I have to know all of them? No, only NOT, AND, OR

The three correspond to negation (¬), conjunction (∧) and disjunction (∨).

AND and **OR**

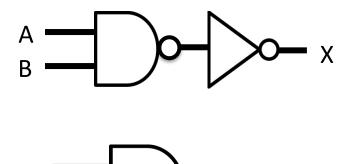
► We can build more functions using the known gates

Α	В	X = A AND B
0	0	0
0	1	0
1	0	0
1	1	1

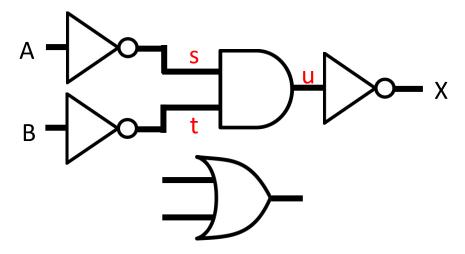
Α	В	A OR B
0	0	0
0	1	1
1	0	1
1	1	1

S	t	u
1	1	1
1	0	0
0	1	0
0	0	0

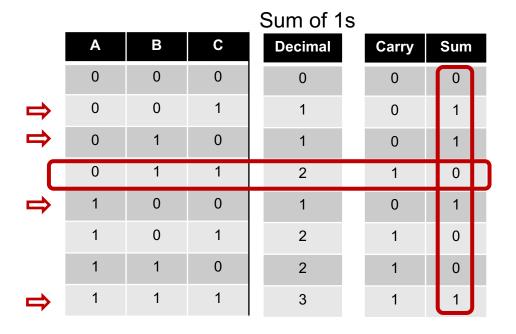
A AND B = NOT (A NAND B)







Circuits Can Implement Arbitrary Functions



```
Sum = (NOT A AND NOT B AND C) OR (NOT A AND B AND C) OR (NOT A AND NOT B AND NOT C) OR (A AND NOT B AND NOT C) OR (A AND B AND C) OR (A AND B AND C)

Carry = (NOT A AND B AND C) OR (B OR C)
```

Example

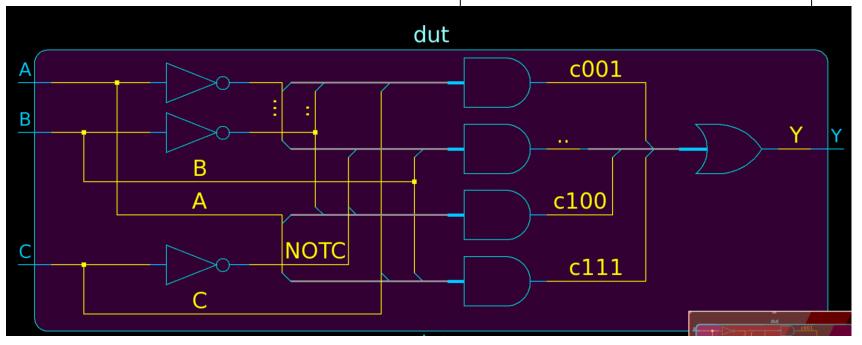
```
Y = (NOT A AND NOT B AND C) OR
(NOT A AND B AND NOT C) OR
( A AND NOT B AND NOT C) OR
( A AND B AND C)
```

In Verilog (a hardware description language)

```
not n1(NOTA,A);
not n2(NOTB,B);
not n3(NOTC,C);

and a1(c001, NOTA, NOTB, C);
and a2(c010, NOTA, B, NOTC);
and a3(c100, A, NOTB, NOTC);
and a4(c111, A, B, C);

or o1(Y, c001, c010, c100, c111);
```



Adding is simple...

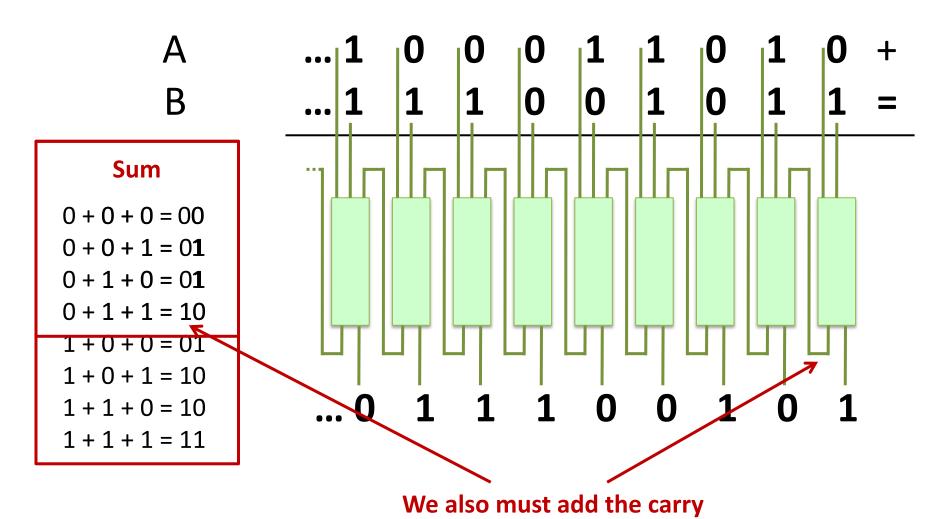
```
A 0111010101100011010 +
B 101110001011100101 =

11100011110001101

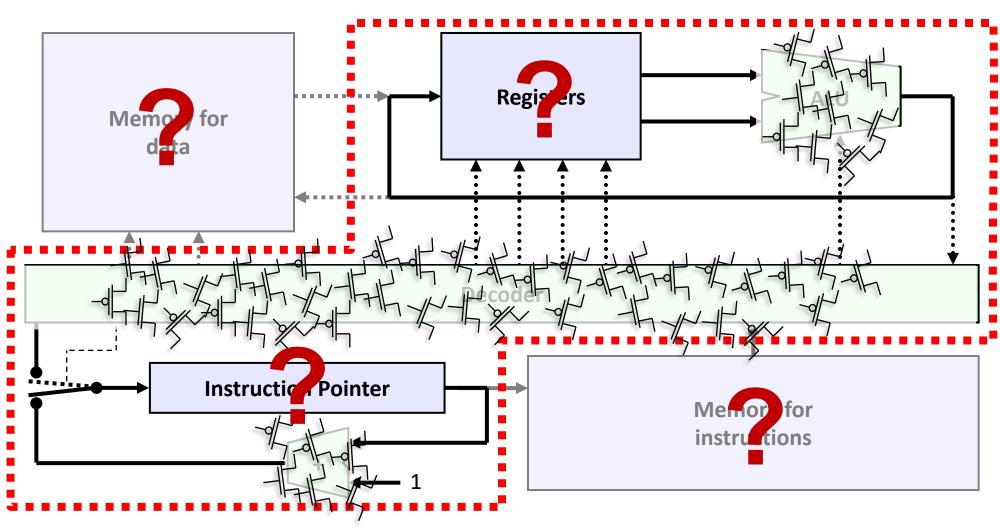
10010111000011101

Carry
```

Making an Adder is simple...

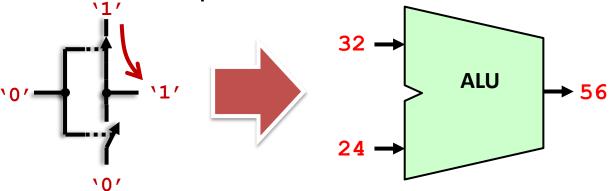


What about Registers and Memory?

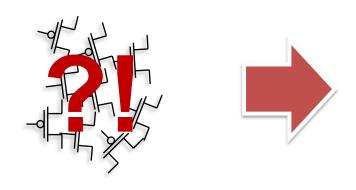


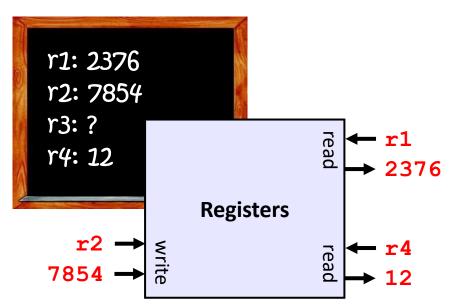
How to Store Information?

➤ We know now how to compute:



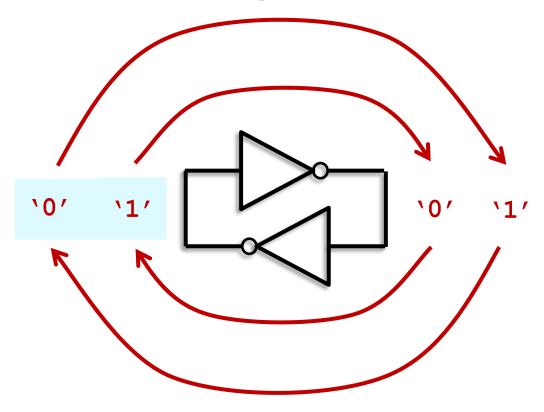
How can we store the result?!





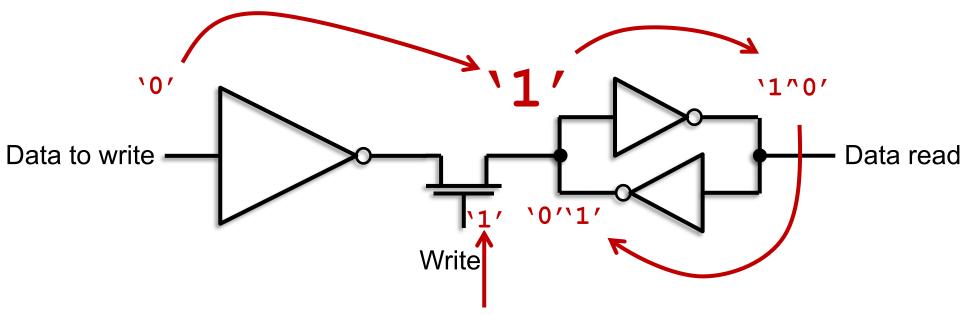
55

A Rather Special Circuit

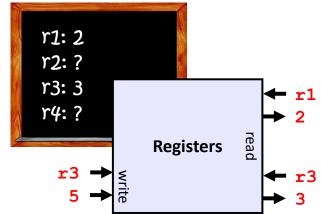


➤ A "bistable" circuit, i.e., it can be in one of two perfectly stable states. A memory element of 1 bit!

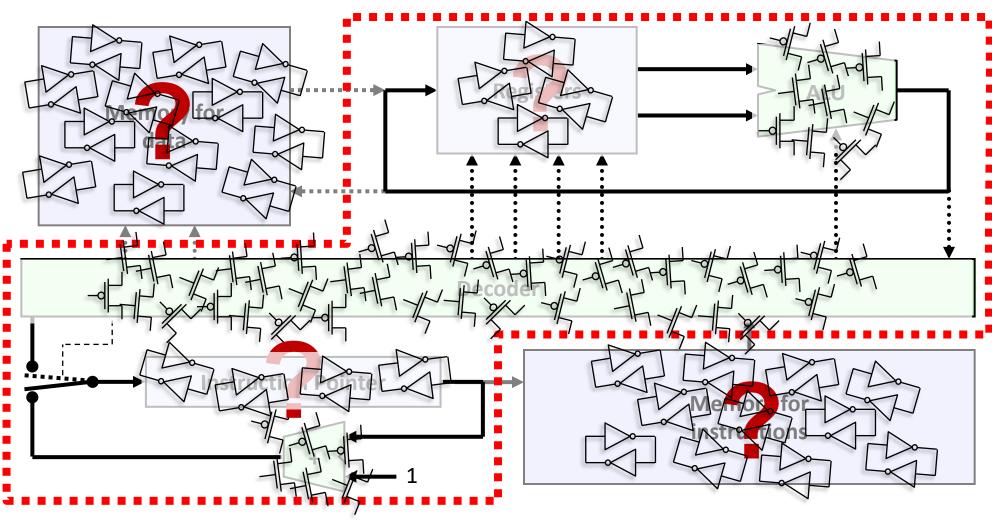
How to Write in this Memory?

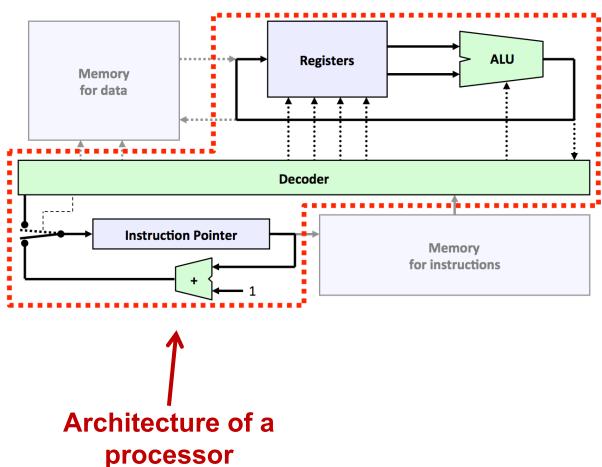


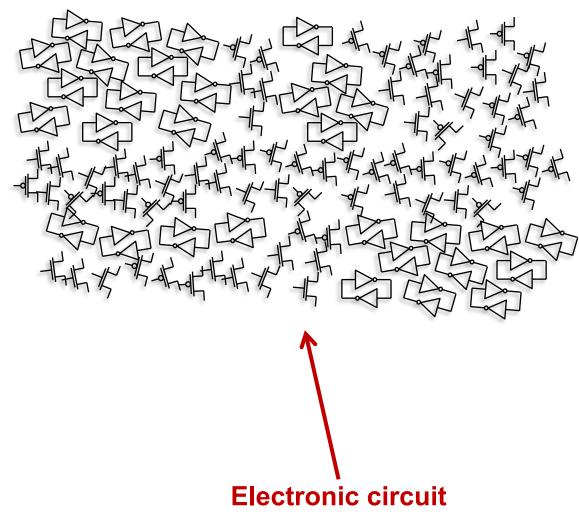
With just a few transistors, we get a perfect memory circuit for all the bits of our registers and memories

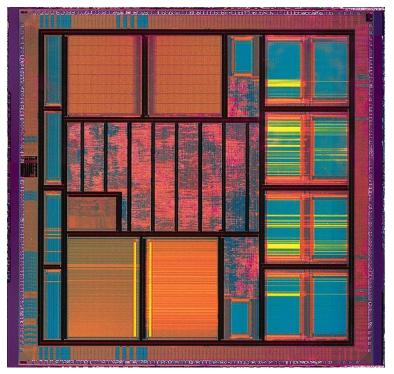


What about Registers and Memory?



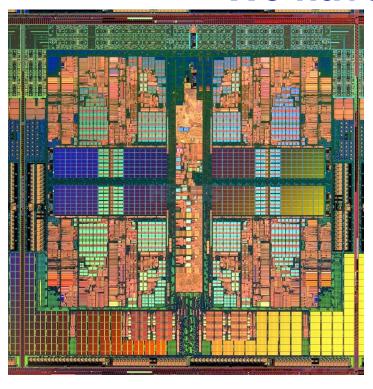






A VLSI circuit can have nowadays around several 10⁹ transistors

- ➤ Transistors can be extremely cheap and small:
 - A transistor costs as little as 10⁻⁸ and 10⁻⁷ cents (CHF/USD/EUR)
 - Modern device consists of Billion of transistors, e.g., Apple A17 SoC using in iPhone 15 Pro has about 19 Billion (10⁹) transistors on about 100mm²





From Algorithms to Computers

sum of first *n* integers

Input: n Output: m

 $s \leftarrow 0$ while n > 0 $s \leftarrow s + n$

 $n \leftarrow n - 1$

 $m \leftarrow s$

Programming Langages



C, C++, C#, Java, Scala, Python, Perl, PHP, SQL, Excel, etc.



sum of first *n* integers

Input: r1 Output: r2

1: copy r3, 0 2: jump egz r1, 6 3: add r3, r3, r1

4: add r1, r1, -1

5: jump 2

6: copy r2, r3



sum of first n integers

Input: r1 Output: r2

1: 0100010100000000

2: 0101100000001010 3: 0001001001011010

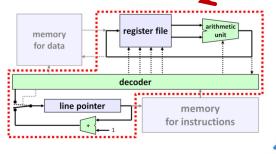
4: 0001010101000000

5: 0000101010101111

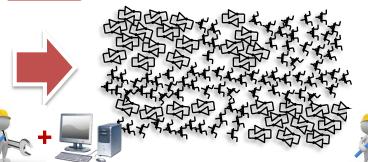
6: 0101000001100101

Software

Hardware













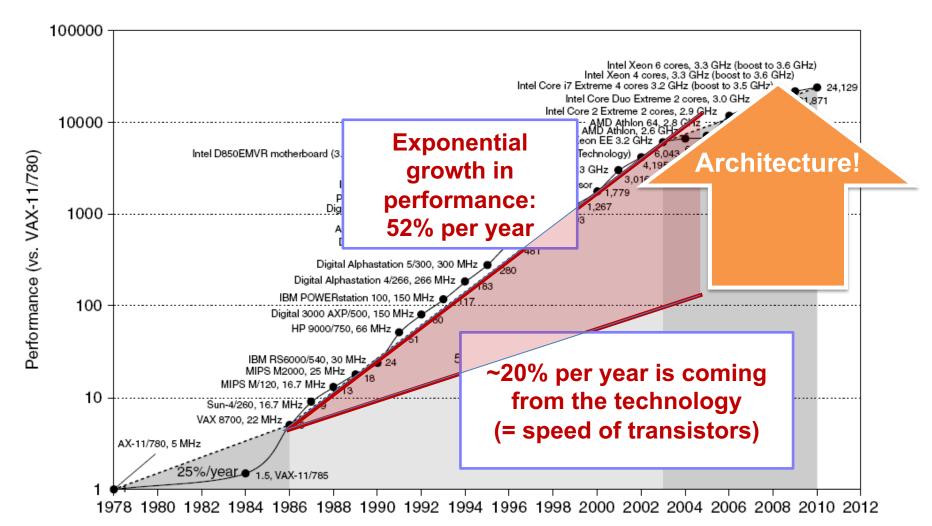


Summary

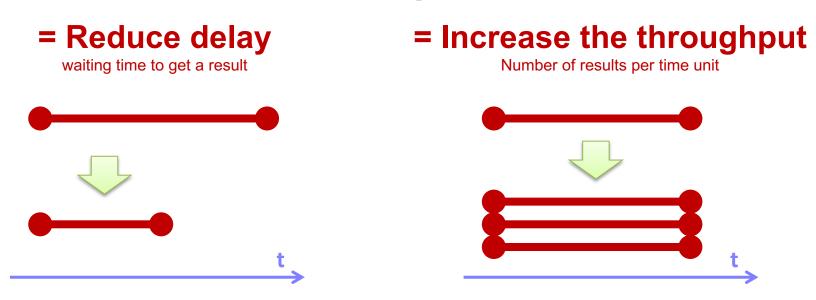
- ► From Algorithms to Computers
 - Assembly code = Basic instructions
 - Processor structure (registers, ALU, instruction counter, memories)
 - Transistors use to compute and store

Question

► How can we make these systems **faster**?



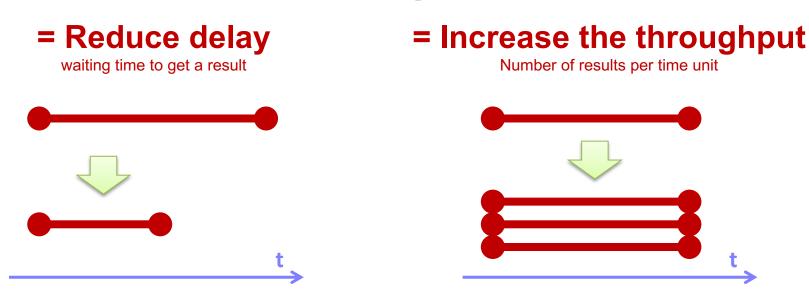
How to increase performance?



Two simple examples to improve performance:

- At the level of the circuit: reduce the delay of an adder
- 2. At the level of the process structure: increase the throughput of instructions

How to increase performance?



Two simple examples to improve performance:

- At the level of the circuit: reduce the delay of an adder
- 2. At the level of the process structure: increase the throughput of instructions

Adding is simple...

B 1011100010111001011 =

 $111000111100011010 \leftarrow$

10010111000011100101

Carry

Elementary sums:

$$0 + 0 = 0$$

 $0 + 1 = 1$
 $1 + 0 = 1$
 $1 + 1 = 10 = 1 \cdot 2^{1} + 0 \cdot 2^{0} = 2_{10}$

Making an Adder is simple...

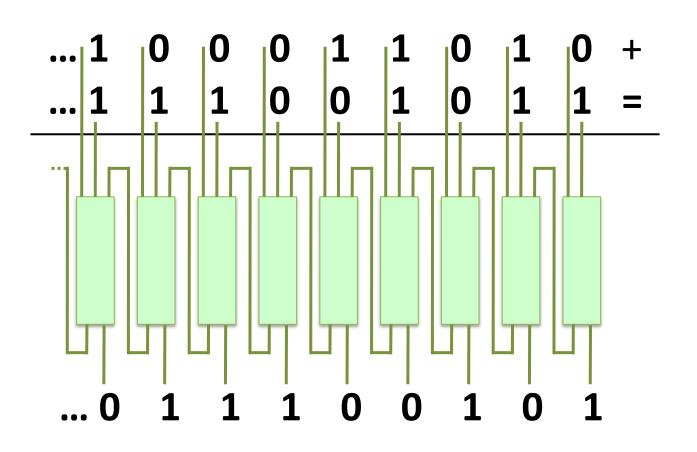
A

B

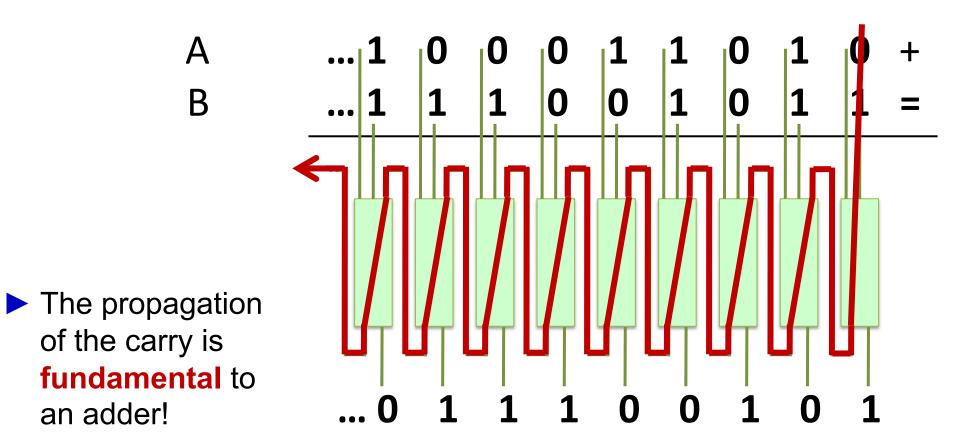
Sum

$$0 + 0 + 0 = 00$$

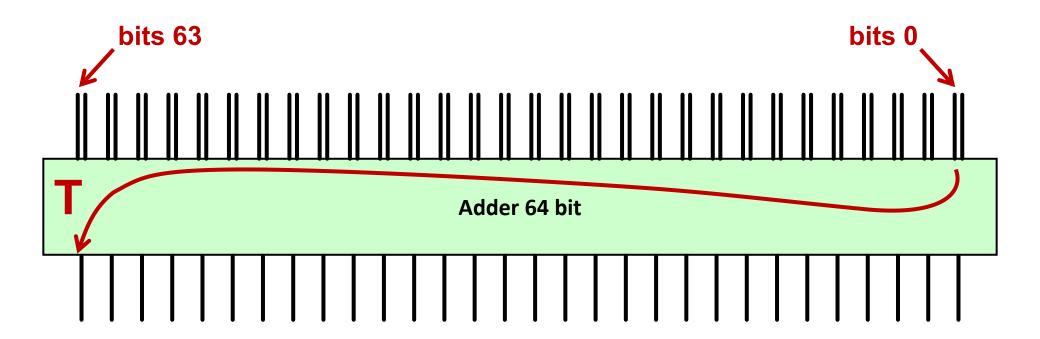
 $0 + 0 + 1 = 01$
 $0 + 1 + 0 = 01$
 $0 + 1 + 1 = 10$
 $1 + 0 + 0 = 01$
 $1 + 0 + 1 = 10$
 $1 + 1 + 0 = 10$
 $1 + 1 + 1 = 11$

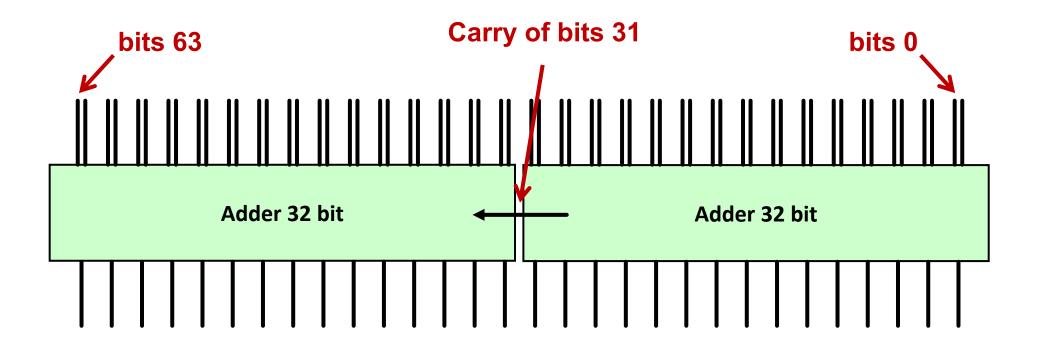


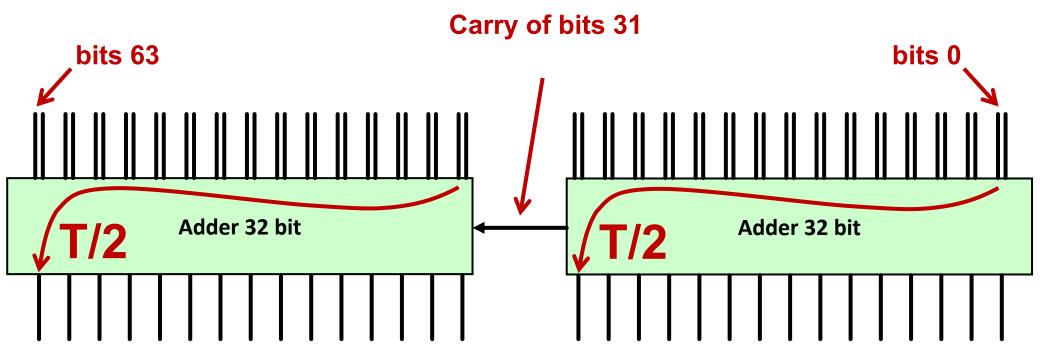
But this circuit is slow



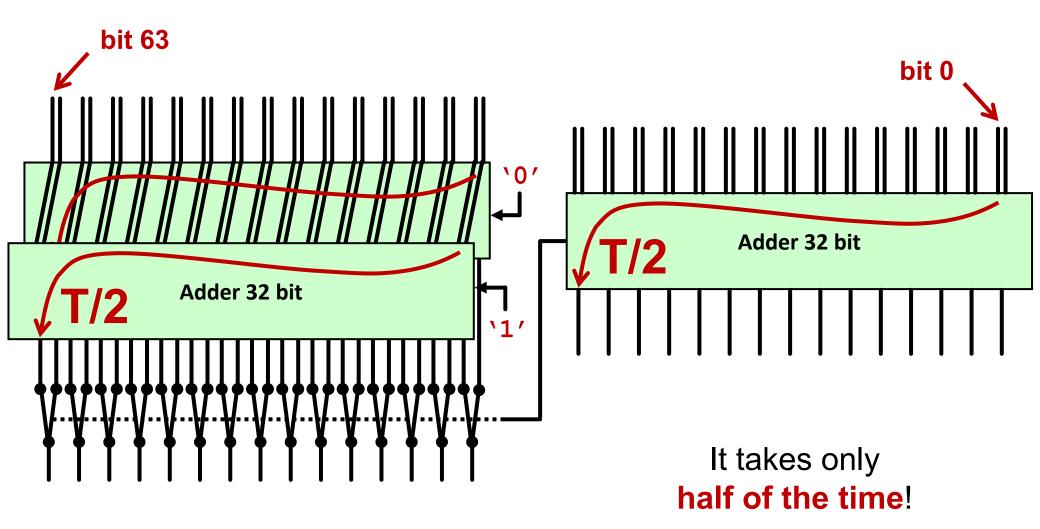
By default, the delay of an adder is proportional to the number of bits.







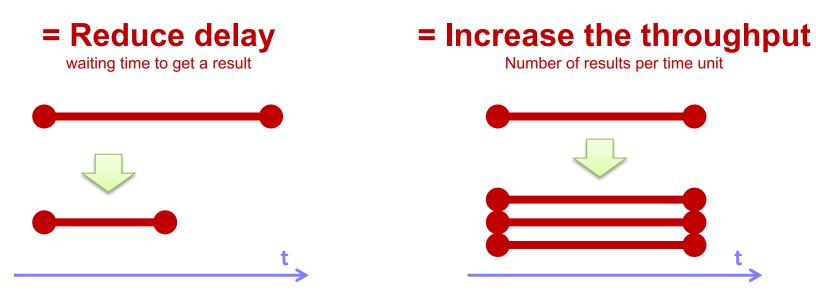
We did not win anything...



Computer Engineering

- We can change the performance of a circuit without changing the functionality.
- We can invest more transistors and more energy to get faster circuits.
- We can slow down circuits to save energy.

How to increase performance?



Two simple examples to improve performance:

- At the level of the circuit: reduce the delay of an adder
- 2. At the level of the process structure: increase the throughput of instructions

Our Processor

```
103: copy r1, 0
104: copy r2, -21
105: sum r3, r7, r4
106: multiply r2, r5, r9
107: subtract r8, r7, r9
108: copy r9, r4
109: sum r3, r2, r1
110: subtract r5, r3, r4
111: copy
            r2, r3
112: sum r1, r2, -1
113: sum r8, r1, -1
114: divide r4, r1, r7
115: copy r2, r4
```

By default, we execute one instruction at a time

```
copy r9, r4
subtract r8, r7, r9
multiply r2, r5, r9
sum r3, r7, r4
copy r2, -21
copy r1, 0

ALU
```

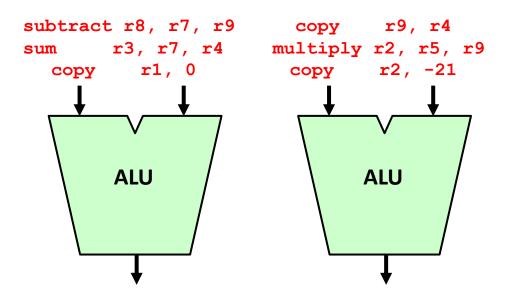
Can we do better?

Double Throughput of Processor

```
103: copy r1, 0
104: copy r2, -21
105: sum r3, r7, r4
106: multiply r2, r5, r9
107: subtract r8, r7, r9
108: copy r9, r4
109: sum r3, r2, r1
110: subtract r5, r3, r4
111: copy r2, r3
112: sum r1, r2, -1
113: sum r8, r1, -1
114: divide r4, r1, r7
115: copy r2, r4
```

Problem?!

One could execute two instructions at a time!



Double Throughput of Processor

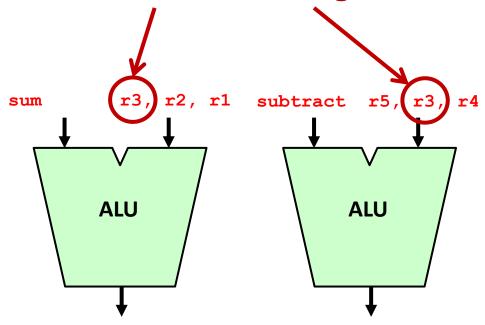
```
103: copy r1, 0
104: copy r2, -21
105: sum r3, r7, r4
106: multiply r2, r5, r9
107: subtract r8, r7, r9
108: copy r9, r4
109: sum r3, r2, r1
110: subtract r5, r3, r4
111: copy r2, r3
112: sum r1, r2, -1
113: sum r8, r1, -1
114: divide r4, r1, r7
115: copy r2, r4
```

Problem?!

A problem occurs when the second instruction needs the result of the first computation!

If you are not careful,

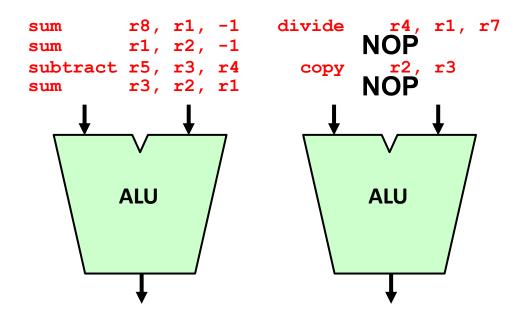
the result is wrong!



Double Throughput of Processor

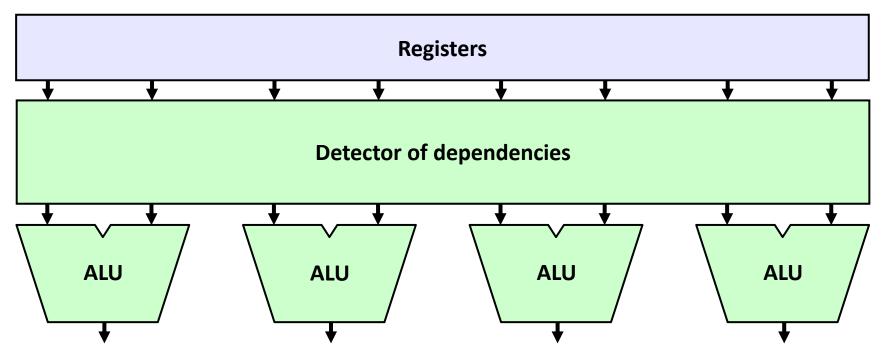
```
r1, 0
103: copy
104: copy r2, -21
105: sum r3, r7, r4
106: multiply r2, r5, r9
107: subtract r8, r7, r9
108: copy r9, r4
109: sum r3, r2, r1
110: subtract r5, r3, r4
111: copy r2, r3
112: sum r1, r2, -1
113: sum r8, r1, -1
114: divide r4, r1, r7
115: copy r2, r4
```

In pratice, one execute between one and two instructions at a time and the result is correct!



NOP ... no operation

A Superscalar Processor



- ➤ All modern processors for laptops, phone, tablets, and for servers are of this type
- In addition, they reorder and execute the instructions before knowing if these instructions will be executed at all (e.g., they might be skipped due to an instruction like jump)

Computer Engineering

- We can change the system structure to run programs faster.
- We can add recourses to the processors to make them faster.
- We can use very basic processors to make them economical and energy efficient.

Summary

- ► From Algorithms to Computers
 - Assembly code = Basic instructions
 - Processor structure (registers, ALU, instruction counter, memories)
 - Transistors use to compute and store
- Performance
 - Reduce the delay of an adder
 - Increase the throughput of instructions