Information, Computation, and Communication

Module Communication

Introduction

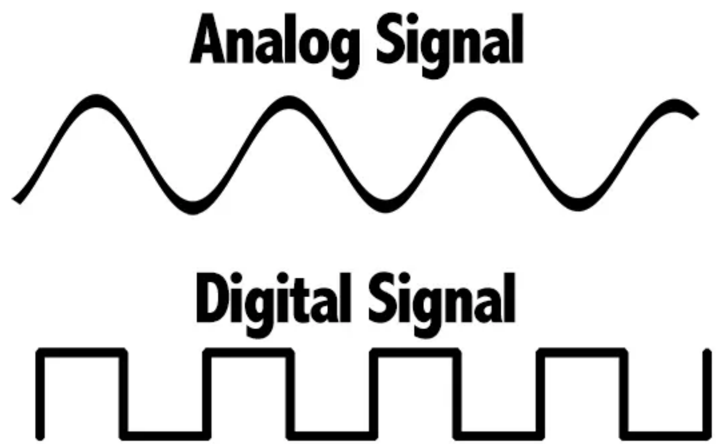
Introduction

- Assume that you have friends that lives in New-Zealand
- You would like to record a video and send it to them
- Nowadays it is possible to accomplish this task within a few minutes
- What is happening exactly during this task?

Introduction

- With your smartphone, you will record a video (image and sound)
  - During this process, an **analog signal** is converted into its **digital representation** with the help of a **sampling algorithm**
  - In addition, another algorithm is used to save the data into a file in the storage.

[by Ivan Radic]



Introduction

- Then you are going to upload your video to your preferred website but first, most likely, you will reduce its size using a **compression algorithm**, so that the upload does not take too long.
  - During the upload, two **error correcting algorithms** will protect the transmission of your data
    (a) on the WIFI network and (b) on the Internet
  - If you do not want other users to see your sketch, an **encryption algorithm** can be used to avoid that other users can access it.

Introduction

- Finally, your friends will be notified that you provided a video. Now they can download and watch your video.
  - During this step, they will use again an **error-correcting algorithm** (and potentially a decryption algorithm) to access the video.
  - The video signal is then **reconstructed from its digital representation**

Outline of this Module

- Topics of the module
  - **Sampling and reconstruction** of a signal
  - **Compression** of data

- We are not going to discuss
  - Data transmission and **error correction**
  - **Encryption/Decryption**

Communication Science

Questions in this Module

- During this module, we will aim to answer the following questions:

- How can we represent the physical reality with bits?
- How can we reconstruction this reality from the partial information (store in these bits) ?
- How can we measure the information stored in some data?
- How can we store some information using the smallest amount of storage (space)?

Sampling and Reconstruction

Outline

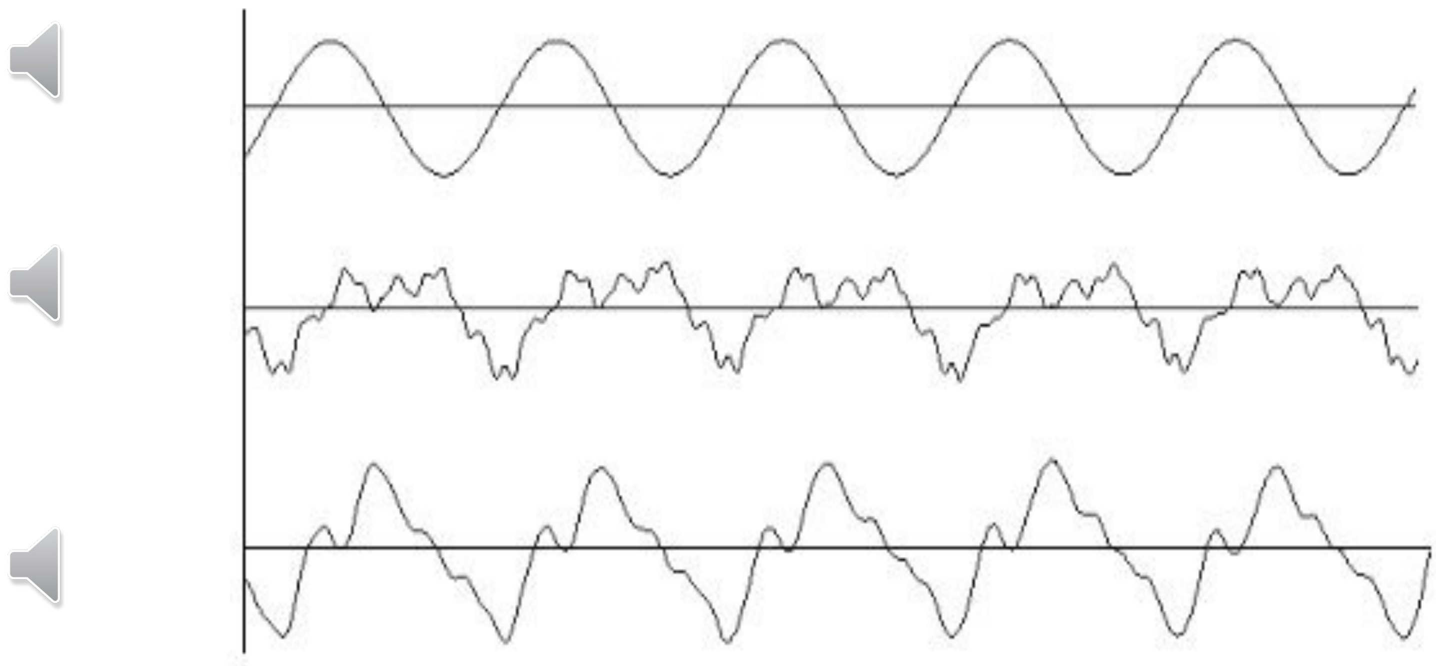
- Sampling and Reconstruction
  - Signal, Frequencies, Spectrum, Bandwidth
  - Filtering
  - Sampling

  (Next week)
  - Reconstruction
  - Sampling Theorem
  - Sub-sampling

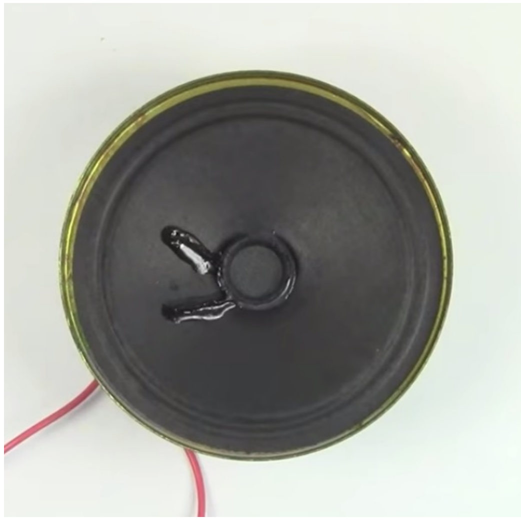
Signals, Frequencies and Bandwidth

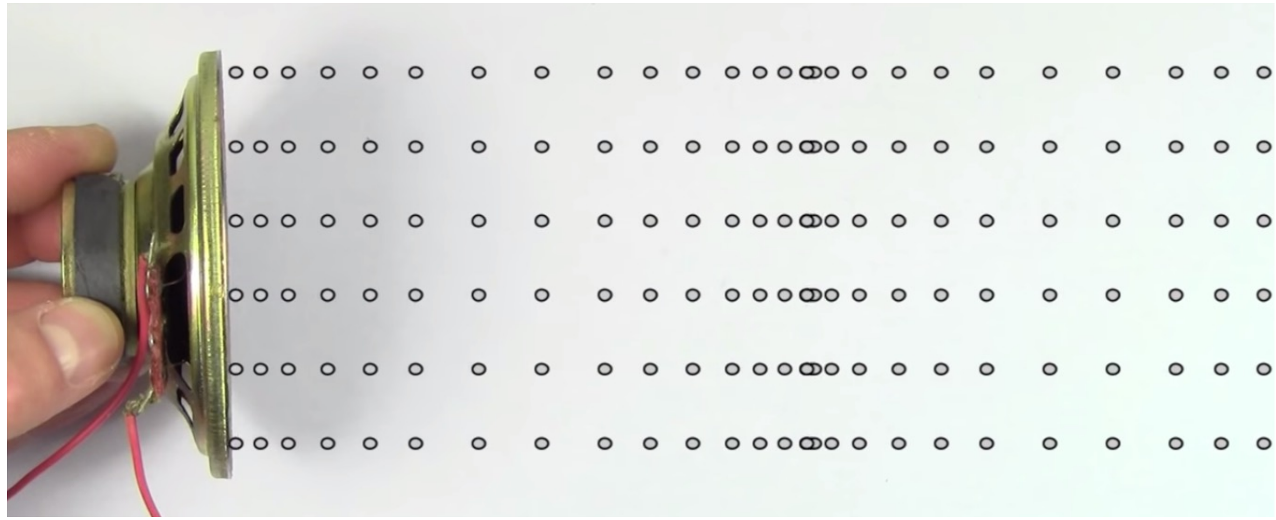
- What is a signal? It is a function
- Examples:

1. Sound wave $X : R \rightarrow R$

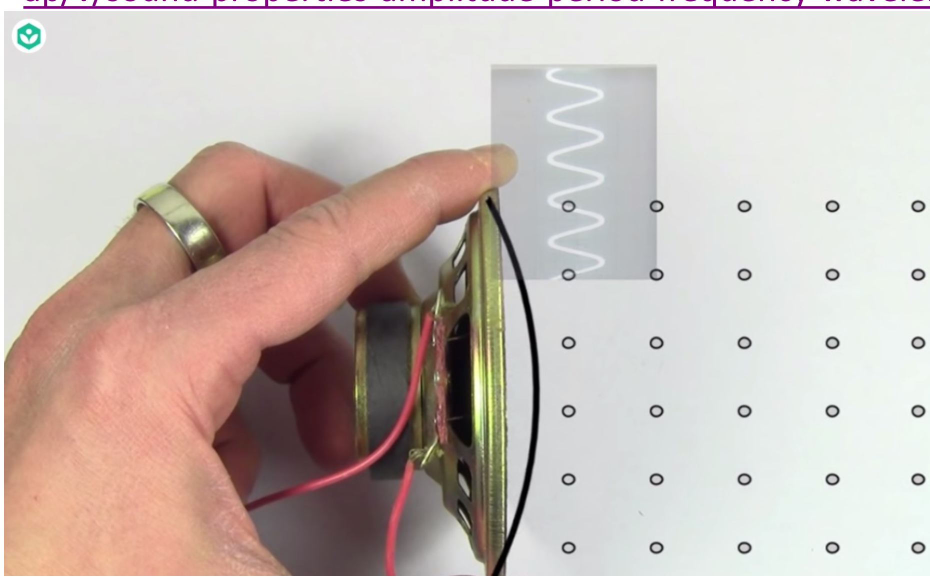


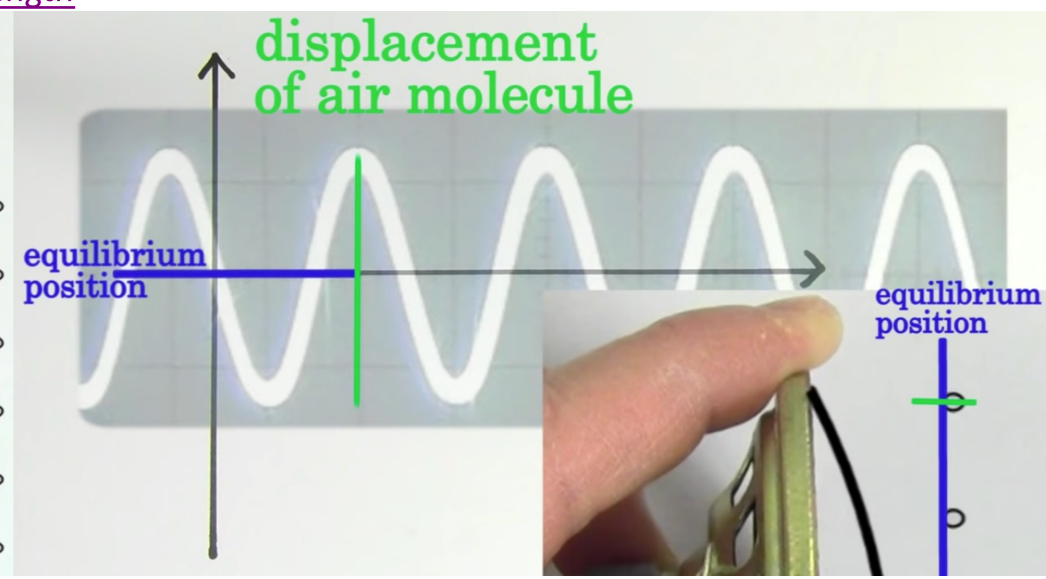
Sounds Waves





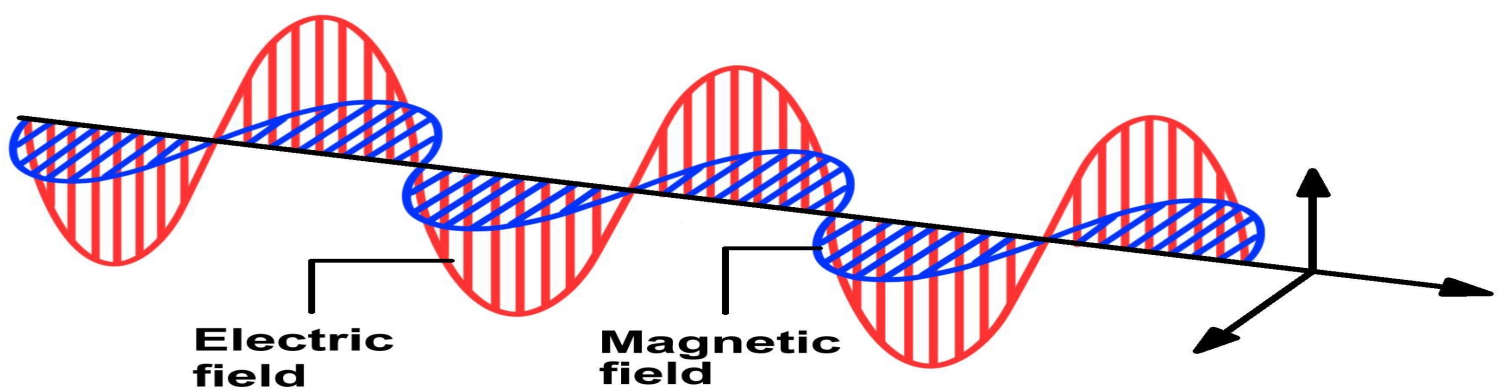
https://www.khanacademy.org/science/ap-physics-1/ap-mechanical-waves-and-sound/introduction-to-sound-waves-ap/v/sound-properties-amplitude-period-frequency-wavelength





Signals, Frequencies and Bandwidth

- What is a signal? It is a function
- Examples:
  1. Sound wave $X : R \rightarrow R$
  2. Electromagnetic wave $X : R \rightarrow R^3$



Signals, Frequencies and Bandwidth

- What is a signal? It is a function
- Examples:
  1. Sound wave $X : R \rightarrow R$
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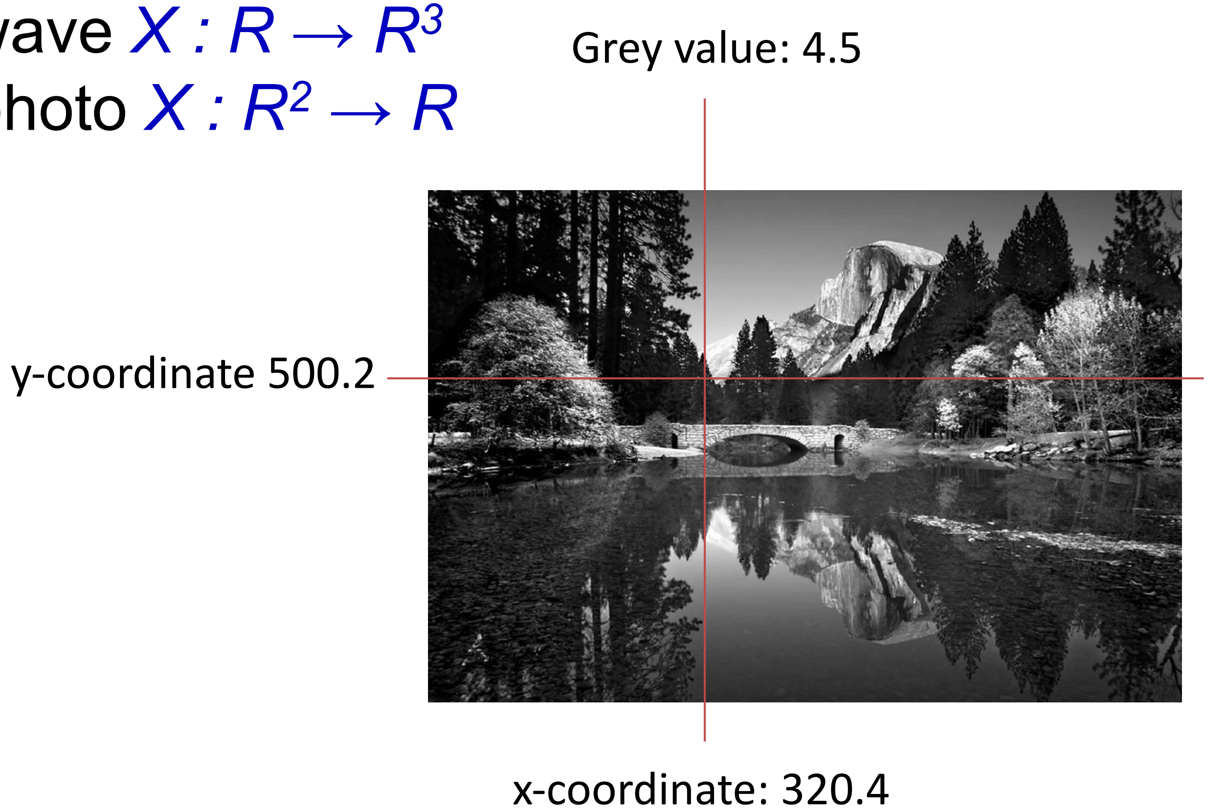
  3. Black and White photo $X : R^2 \rightarrow R$

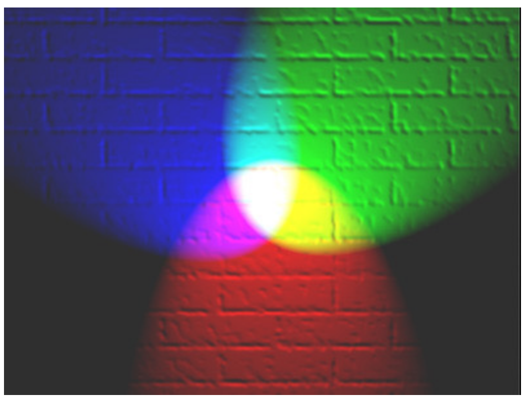


Signals, Frequencies and Bandwidth

- What is a signal? It is a function
- Examples:
  1. Sound wave $X : R \rightarrow R$
  2. Electromagnetic wave $X : R \rightarrow R^3$
  3. Black and White photo $X : R^2 \rightarrow R$
  4. Color photo $X : R^2 \rightarrow R^3$

     $(x, y) \longrightarrow (R,G,B)$

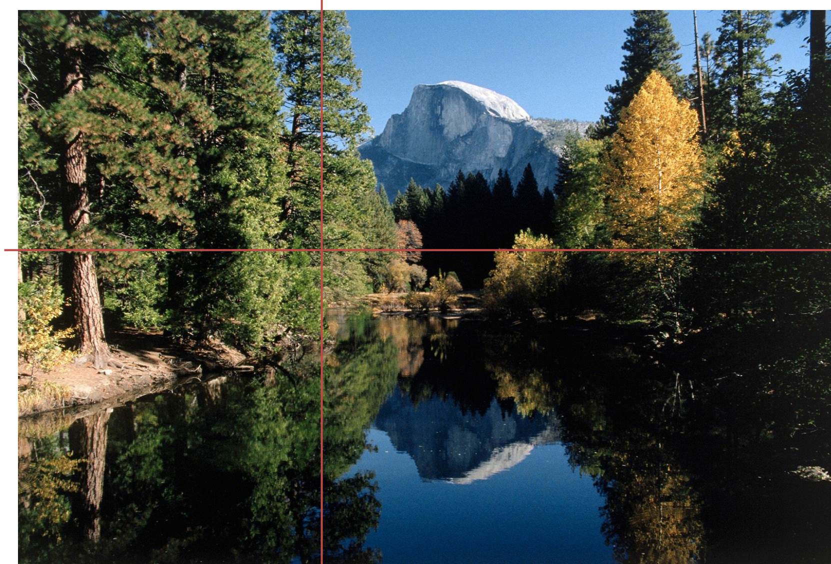
RGB or Additive Color Model



https://en.wikipedia.org/wiki/RGB_color_model

Red: 3.0 Green: 202 Blue: 30

y-coordinate 500.2



x-coordinate: 320.4

Signals, Frequencies and Bandwidth

- What is a signal? It is a function
- Examples:
  1. Sound wave $X : R \rightarrow R$
  2. Electromagnetic wave $X : R \rightarrow R^3$
  3. Black and White photo $X : R^2 \rightarrow R$
  4. Color photo $X : R^2 \rightarrow R^3$
  5. Video $X : R^3 \rightarrow R^3$

     (x, y, time) $\longrightarrow$ (R,G,B)

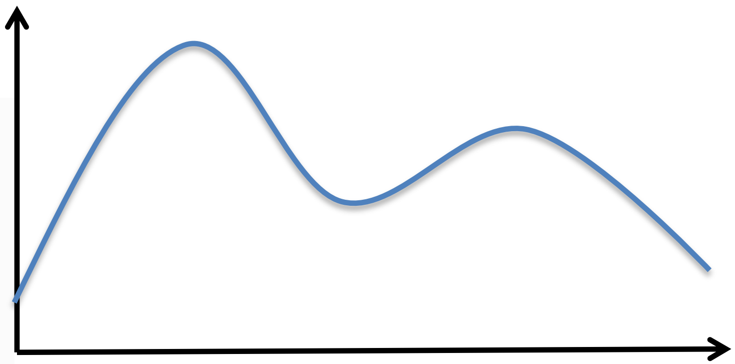
Signals, Frequencies and Bandwidth

- What is a signal? It is a function
- Examples:
  1. Sound wave $X : R \rightarrow R$
  2. Electromagnetic wave $X : R \rightarrow R^3$
  3. Black and White photo $X : R^2 \rightarrow R$
  4. Color photo $X : R^2 \rightarrow R^3$
  5. Video $X : R^3 \rightarrow R^3$

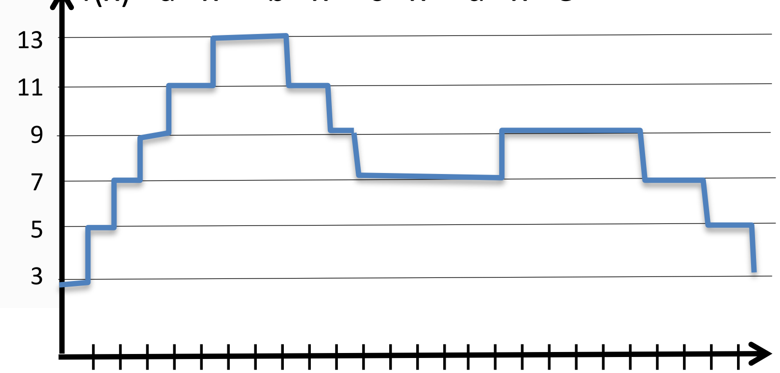
- In general, we can define a signal as a function $X : R^d \rightarrow R^k$
- For clarify and simplicity, we will focus on one-dimensional signals $X : R \rightarrow R$ during this module.

Analog versus Digital Signal

- Analog signals can take arbitrary value from R and can be smooth and continuous.
- Digital signals have a **finite set of possible values and are sampled at discrete time steps.**



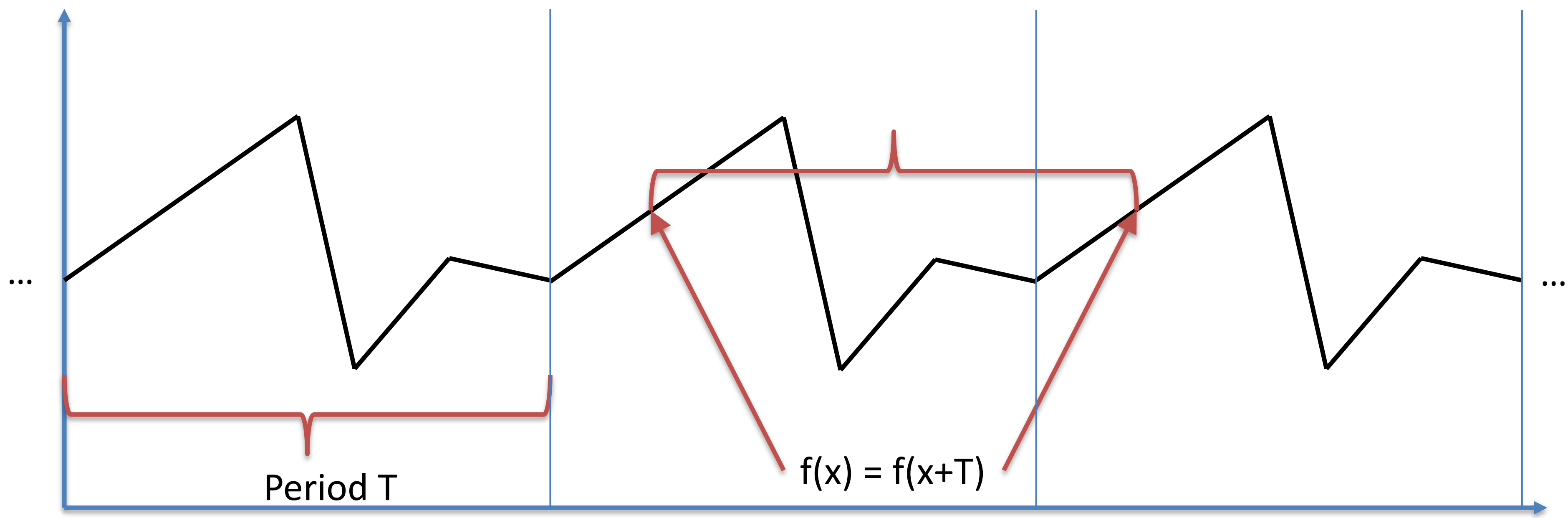
$f(x)= a \cdot x^4 + b \cdot x^3 + c \cdot x^2 + d \cdot x - e$



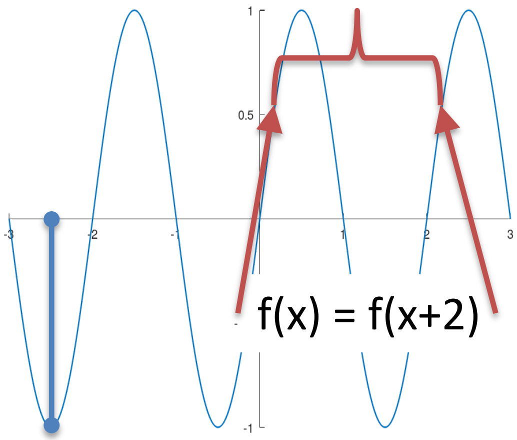
f(n)= {3,5,7,9,11,11,13,13,...}

Periodic Signals

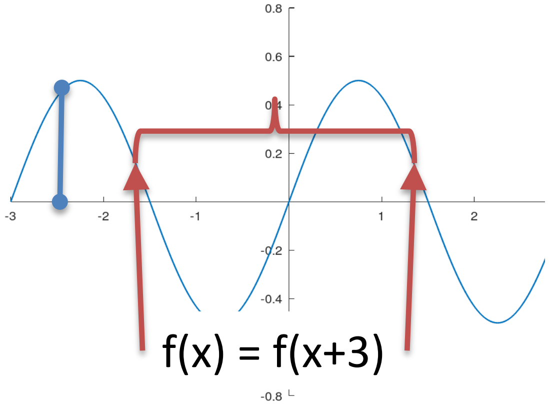
- A **periodic** signal repeats itself after a certain interval.
- Period = the distance between two repetitions
- If a signal is not period is it **aperiodic**.



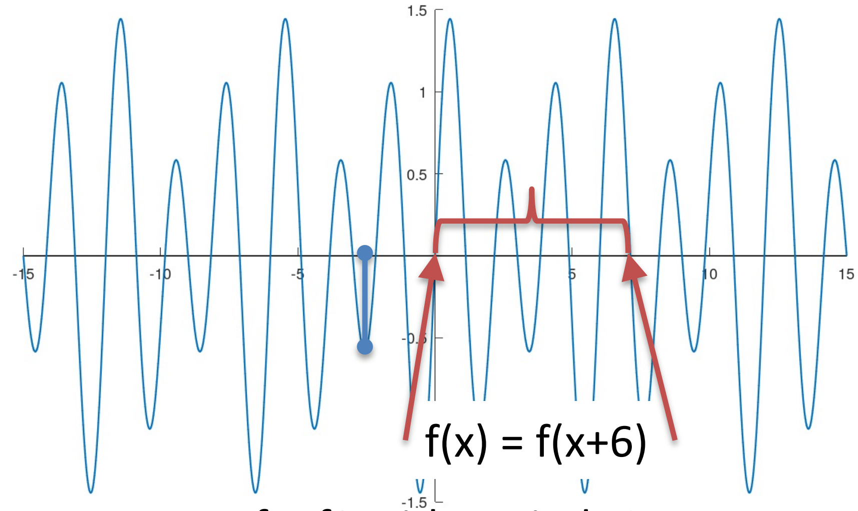
Periodic Signals and their Sums



f1 with period=2



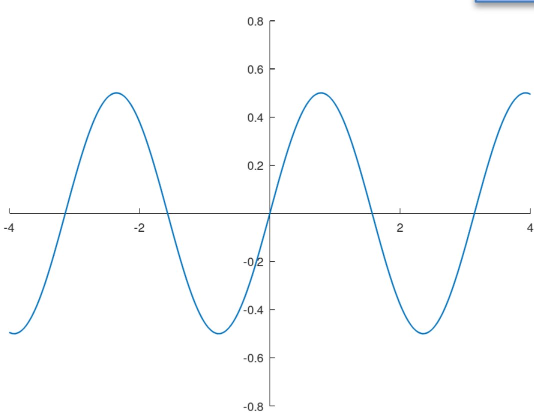
f2 with period=3



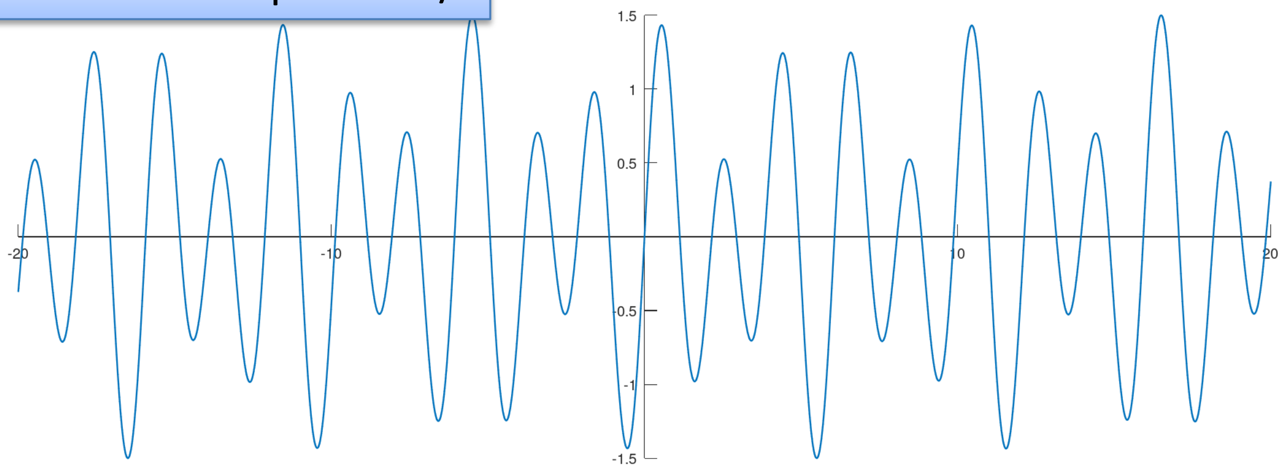
f1+f2 with period=6

$2 \cdot n_1 = 6 \quad 3 \cdot n_2 = 6$

https://www.wolframalpha.com/



f3 with period=π



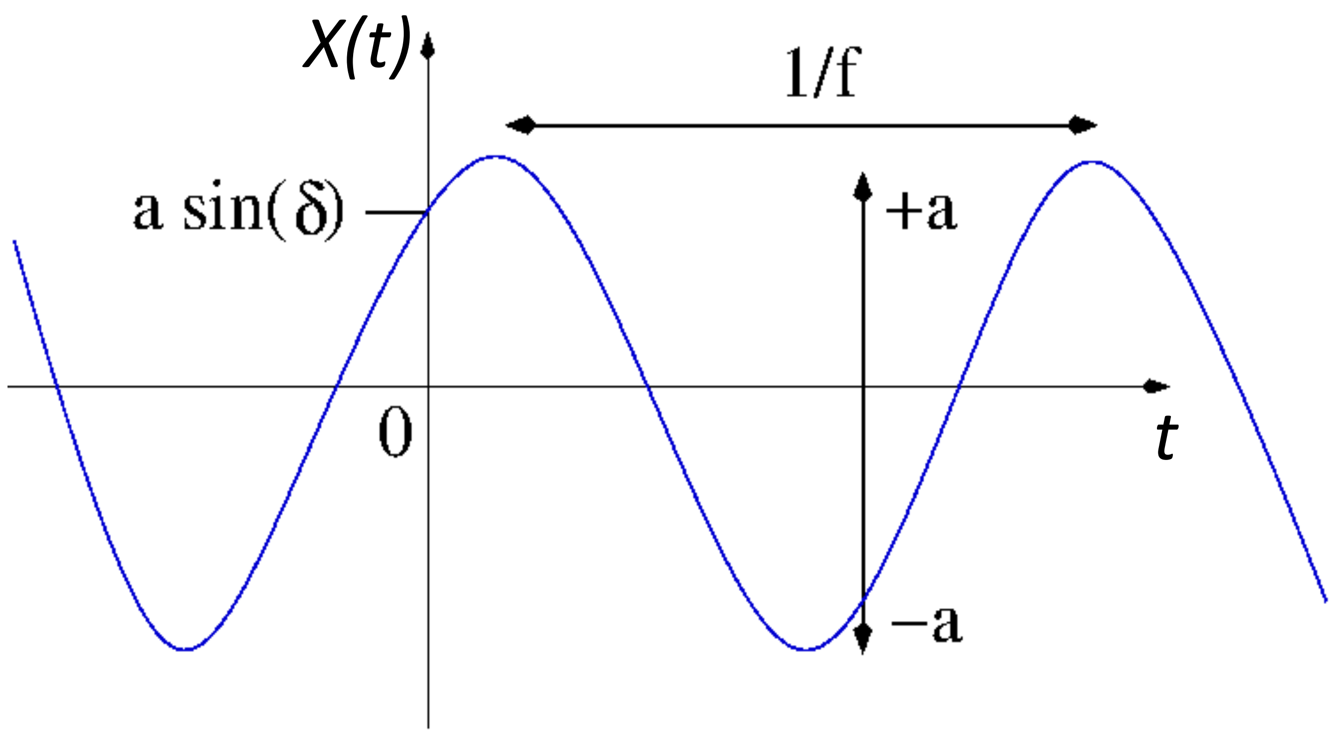
f1+f3 with no period (aperiodic)

Examples of Signals

Sinusoid also called **sine wave:**

$$X(t) = a \sin(2\pi f t + \delta), \quad t \in \mathbb{R}$$

$a =$ amplitude, $f =$ frequency, T = period = 1/f , $\delta =$ phase (shift)



Examples of Signals

Sinusoid:

$$X(t) = a \sin(2\pi f t + \delta), \quad t \in R$$

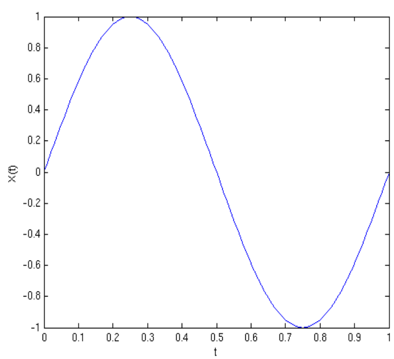
$a =$ amplitude, $f =$ frequency, T = period = 1/f , $\delta =$ phase

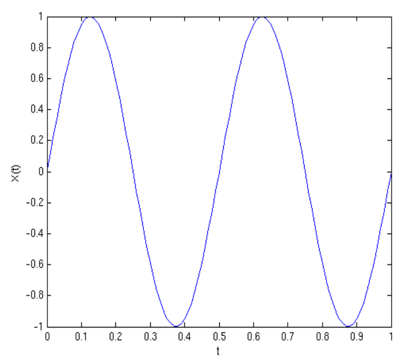
.., $a = 1$, $f = 1$, $\delta = 0$ $X(t) = \sin(2\pi t)$

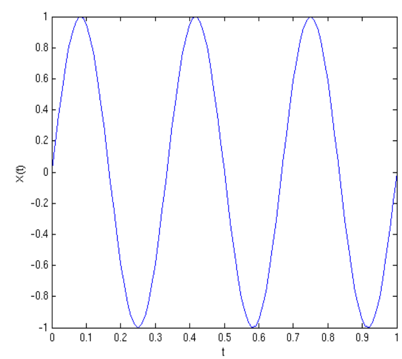
.., $a = 1$, $f = 2$, $\delta = 0$ $X(t) = \sin(4\pi t)$

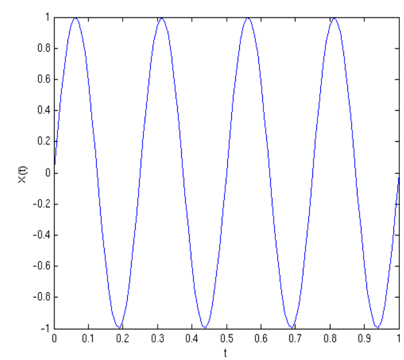
.., $a = 1$, $f = 3$, $\delta = 0$ $X(t) = \sin(6\pi t)$

.., $a = 1$, $f = 4$, $\delta = 0$ $X(t) = \sin(8\pi t)$









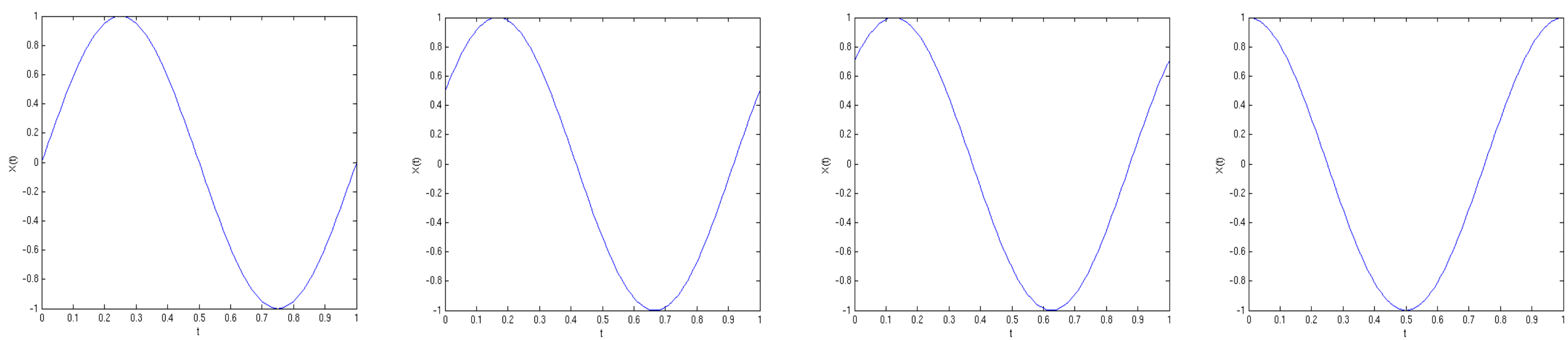
Examples of Signals

Sinusoid:

$$X(t) = a \sin(2\pi f t + \delta), \quad t \in \mathrm{R}$$

$a =$ amplitude, $f =$ frequency, T = period = 1/f, $\delta =$ phase

- $a = 1, f = 1, \quad \delta = 0$ $\qquad X(t) = \sin(2\pi t)$
- $a = 1, f = 1, \quad \delta = \pi/6$ $\qquad X(t) = \sin(2\pi t + \pi/6)$
- $a = 1, f = 1, \quad \delta = \pi/4$ $\qquad X(t) = \sin(2\pi t + \pi/4)$
- $a = 1, f = 1, \quad \delta = \pi/2$ $\qquad X(t) = \sin(2\pi t + \pi/2) = \cos(2\pi t)$



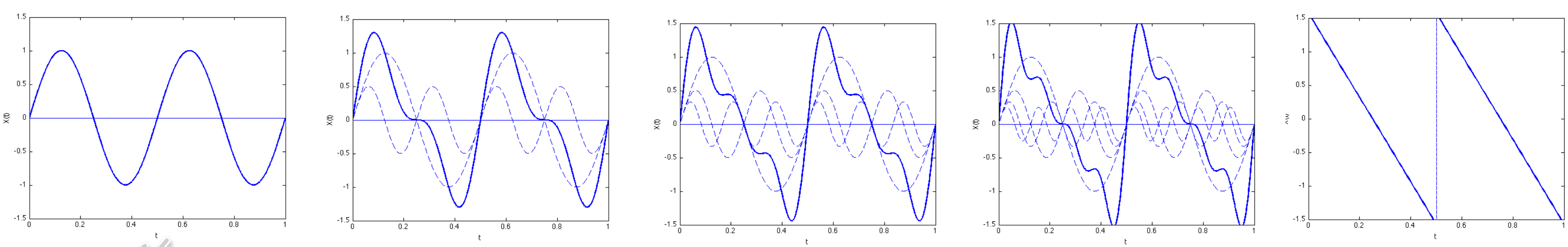
Examples of Signals

Sum of sinusoids:

$$X(t) = a_1 \sin(2\pi f_1 t + \delta_1) + \ldots + a_n \sin(2\pi f_n t + \delta_n), \quad t \in \mathbb{R}$$

a_j = amplitudes, f_j = frequencies, δ_j = phase shift

- Example: $a_j = 1/j$, $f_j = 2j$, $\delta_j = 0$, $n = 1, 2, 3, 4, \ldots$
- $n = 1: X(t) = \sin(4\pi t)$
- $n = 2: X(t) = \sin(4\pi t) + 1/2 \sin(8\pi t)$
- $n = 3: X(t) = \sin(4\pi t) + 1/2 \sin(8\pi t) + 1/3 \sin(12\pi t)$
- $n = 4: X(t) = \sin(4\pi t) + 1/2 \sin(8\pi t) + 1/3 \sin(12\pi t) + 1/4 \sin(16\pi t)$
- "$n = \infty$"



https://en.wikipedia.org/wiki/File:Additive_220Hz_Sawtooth_Wave.wav

Signals in General

- **Statement**:

“All (interesting) signals are sums of sinusoids!”

- In the following, we will consider only signals that are sums of sinusoids.

- More examples: https://en.wikipedia.org/wiki/Fourier_series

Frequencies: Unit

- The frequency f in a sinusoid $X(t) = \boldsymbol{a}\sin(2\pi \boldsymbol{f} t + \boldsymbol{\delta})$ is expressed in hertz = Hz = $\frac{1}{s}$.
- A signal with frequency f Hz repeats every $T = 1/f$ s (seconds)

- Example: the note "La" at 440Hz is a sinusoid that repeats every $\frac{1}{440} = 2.2727...$ milliseconds.

- This unit is named after Heinrich Rudolf **Hertz** (1857-1894), who
  - experimentally verified the Maxwell theory, which proved that light is an electromagnetic wave
  - developed the first system to transmit and receive radio wave.

Frequencies: Orders of Magnitude

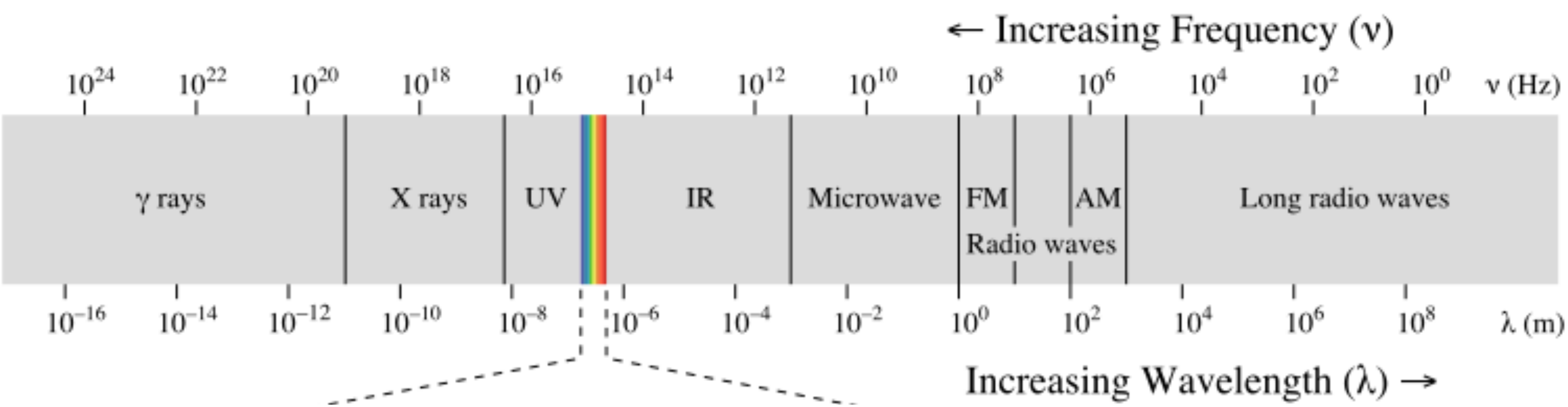
Audio waves:

- 20 Hz - 20 kHz: audible sound
- 20 kHz +: ultra sound

Tone Generator:
http://www.szynalski.com/tone-generator/

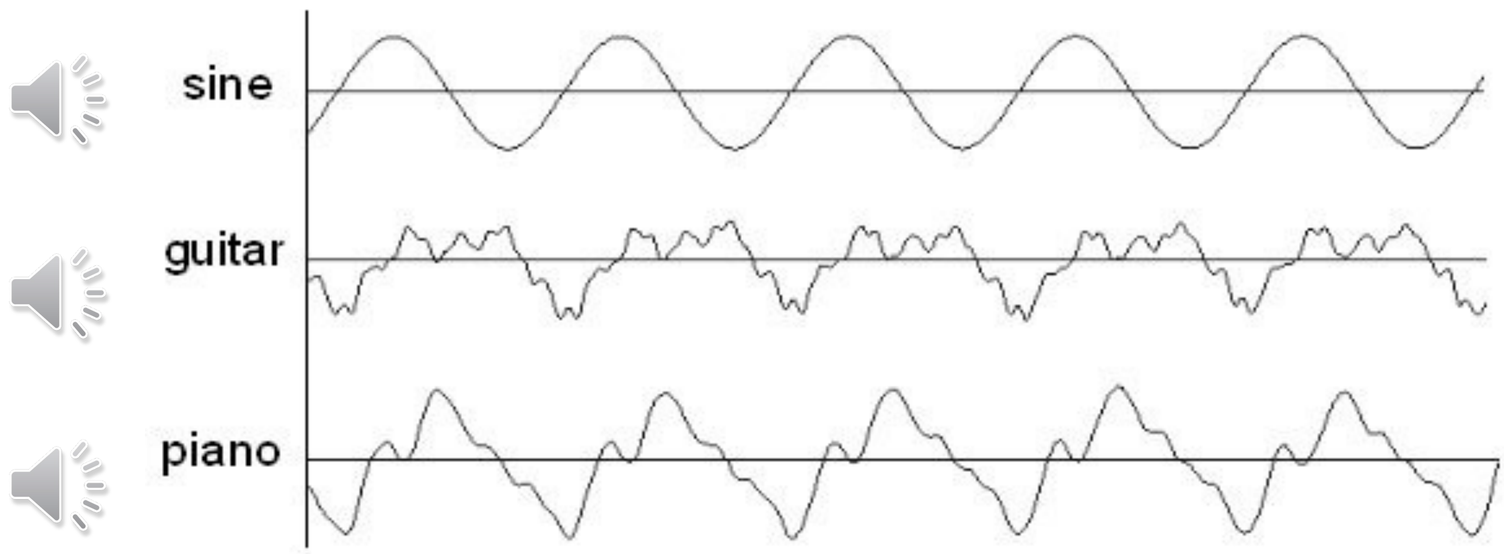
Electromagnetic waves:

- 150 kHz - 3 GHz: radio waves
- 3 GHz - 300 GHz: micro-waves, radar
- 300 GHz - 4.3 x 10^{14} Hz: infra red
- 4.3 x 10^{14} Hz - 7.5 x 10^{14} Hz: visible light
- 7.5 x 10^{14} Hz - 3 x 10^{17} Hz: ultraviolet
- 3 x 10^{17} Hz +: X rays, gamma rays,...



All "La at 440 Hz" are not the same!

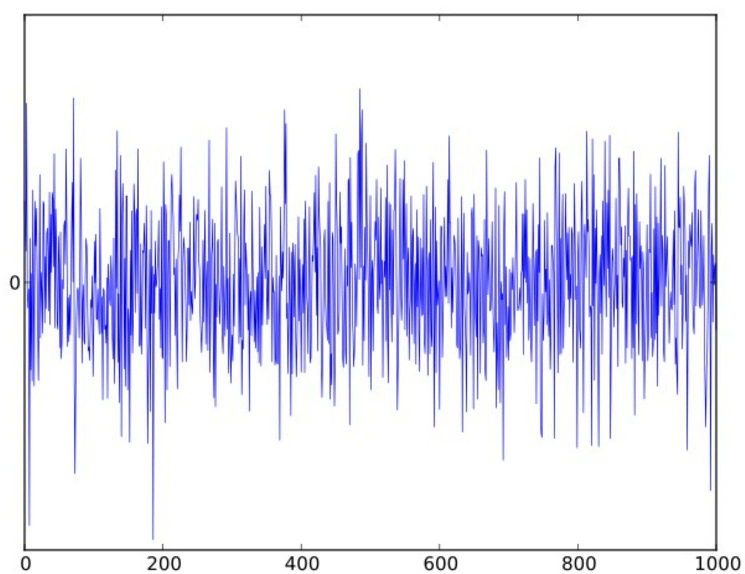
Example from: http://www.yuvalnov.org/temperament/



(If you want to know why a piano has 12 keys in an octave follow the link above.)

Noise

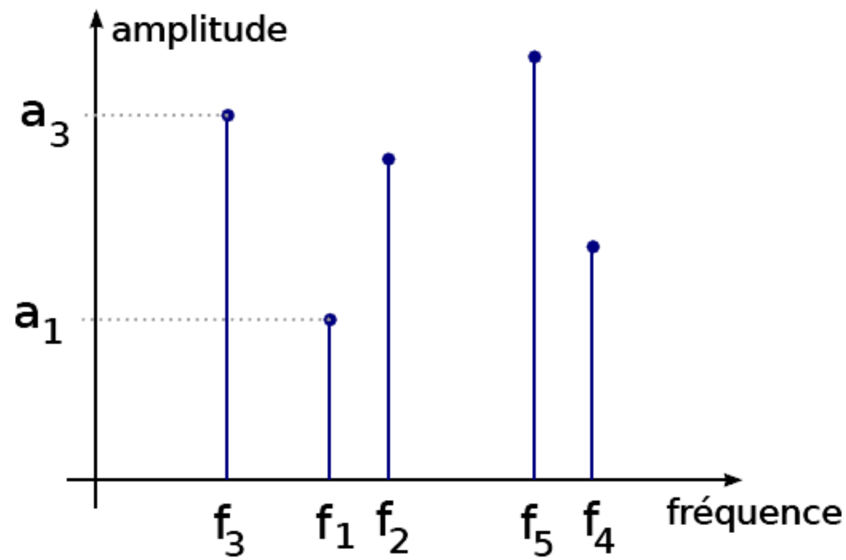
- Sound versus noise (= annoying or unwanted sound or signal)
- White noise is a random signal having equal intensity at different frequencies.



https://en.wikipedia.org/wiki/White_noise

Frequency Spectrum

- In the “frequency space”:
  - Horizontal axis = frequency
  - Vertical axis = amplitude



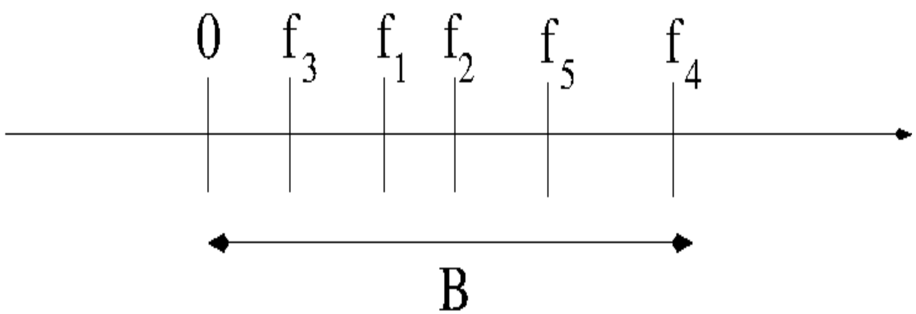
- Example: a sum of sinusoid:
  $X(t) = a_1 \sin(2\pi f_1 t + \delta_1) + \ldots + a_n \sin(2\pi f_n t + \delta_n)$

- This representation is called the **spectrum** of the signal

Bandwidth

- Given a sum of sinusoids:
  $X(t) = a_1 \sin(2\pi f_1 t + \delta_1) + \ldots + a_n \sin(2\pi f_n t + \delta_n)$

- The bandwidth of this signal is defined as follows:
  $B = f_{\max} = \max\{f_1, \ldots, f_n\}$



- As we will see, bandwidth plays a vital role in signal processing.

Filtering a signal

Filtering a signal

- In general, a filter transforms a signal, i.e., if a signal $X(t), t \in R$ passes through a filter, a distorted version $\hat{X}(t), t \in R$ comes out

- Why do we want to filter a signal? Most often, to suppress (or at least reduce) the **noise** present in the signal.
- There are many kinds of filters.
- In this class, we will see a particular category of filters:

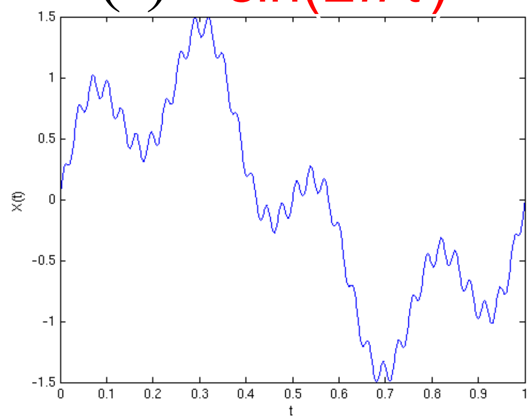
the "**low pass**" filters

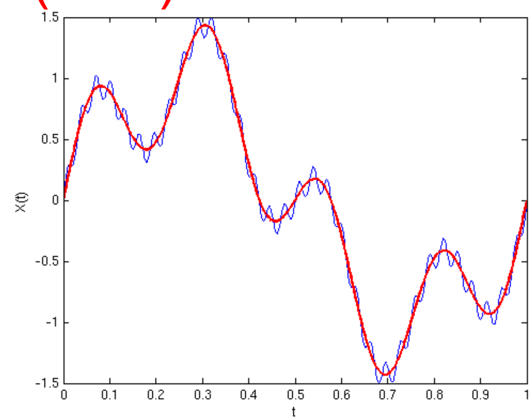
Ideal Low-Pass Filter (Filtre passe-bas idéal)

- An ideal low-pass filter is a filter that **suppresses the high frequencies** that are present in a signal (which are usually the source of noise). It lets the **low** frequencies **pass**!
- More precisely, if X(t) is a sum of sinusoids, then after the filter, all the components of X(t) that have a frequency that is larger than the **cutoff frequency f_c** disappear.
- **Example**: consider the following signal that includes the frequencies 1Hz (A=1), 4Hz (A=1/2), and 32Hz (A=1/10). Assume a filter with f_c=30Hz

$$X(t) = \sin(2\pi t) + 1/2 \sin(8\pi t) + 1/10 \sin(64\pi t)$$

$$\hat{X}(t) = \sin(2\pi t) + 1/2 \sin(8\pi t)$$





Moving Average Filter (Filtre à moyenne mobile)

The output signal $\hat{X}(t)$ at time *t* of a **moving average** filter is given by

$$\hat{X}(t) = \frac{1}{T_c}\int_{t-T_c}^{t} X(s)ds$$

Value at time t is the average of the signal in the interval t - T_c to t

Example: What happens to a sinusoid that passes through such a filter?

$X(t) = \sin(2\pi f t)$ is transformed to

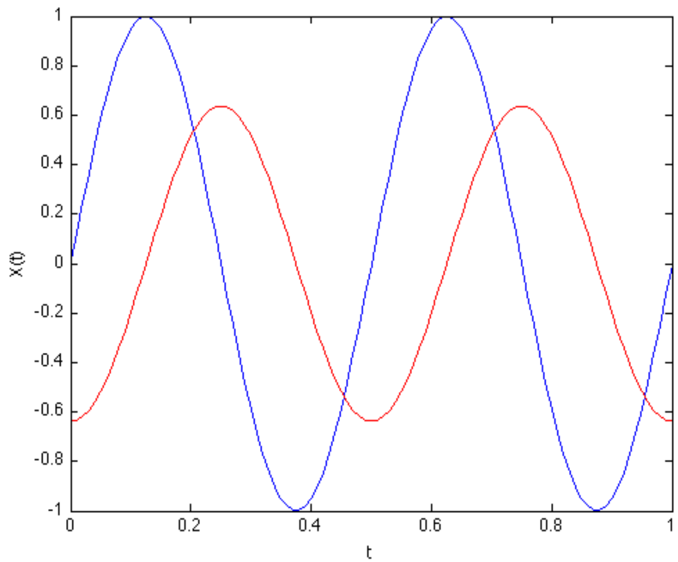
$$\hat{X}(t) = \frac{1}{T_c}\int_{t-T_c}^{t} \sin(2\pi fs)ds$$

$$= \frac{\cos(2\pi f(t-T_c)) - \cos(2\pi ft)}{2\pi fT_c}$$

$$= \frac{\sin(\pi fT_c)}{\pi fT_c}\sin(2\pi ft - \pi fT_c)$$

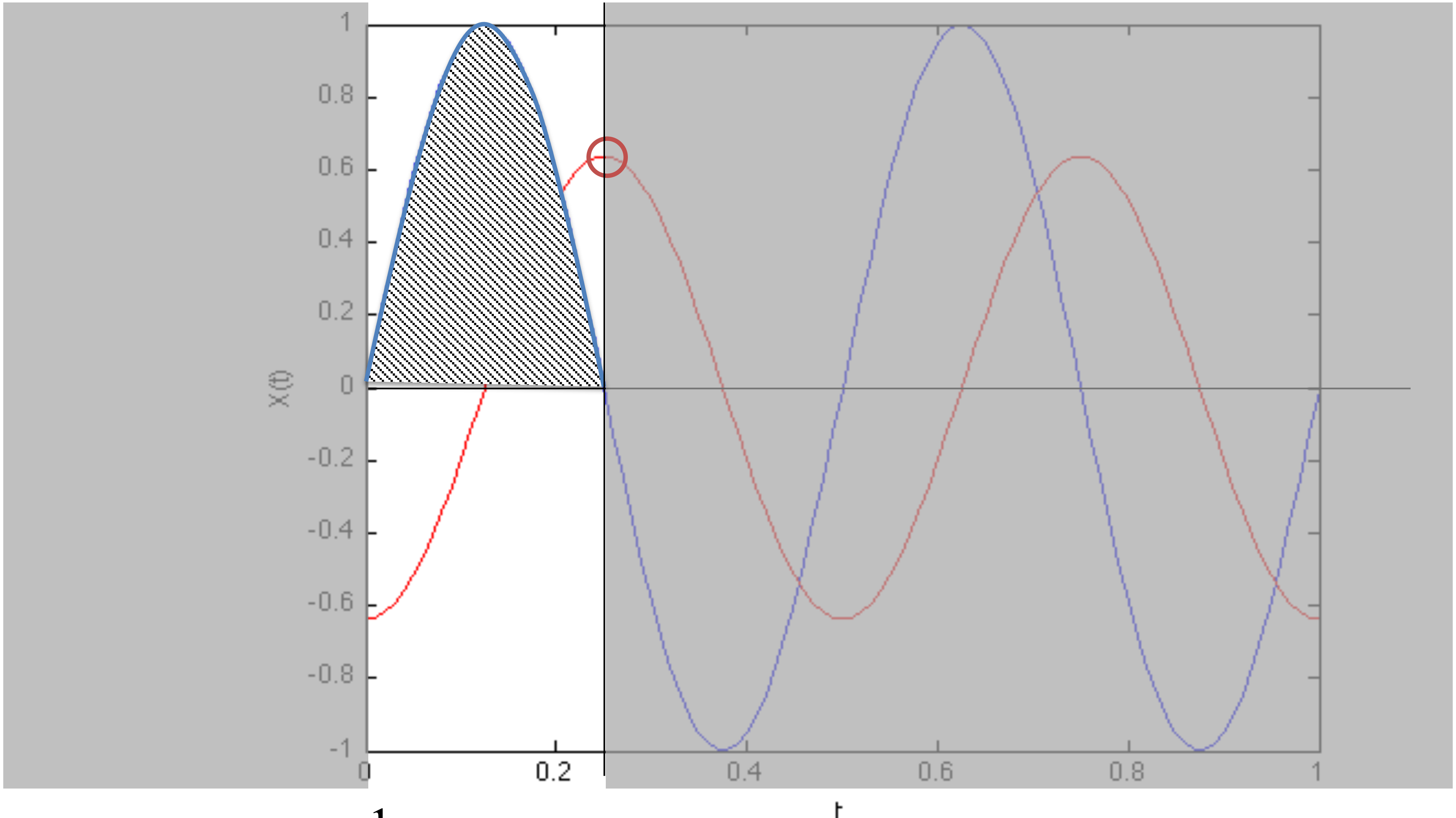
Recall: $\int \sin(a \cdot x)dx = -\frac{1}{a}\cos(a \cdot x)$

Note: $\cos(b) - \cos(a) = 2 \cdot \sin\left(\frac{a+b}{2}\right) \cdot \sin(\frac{a-b}{2})$



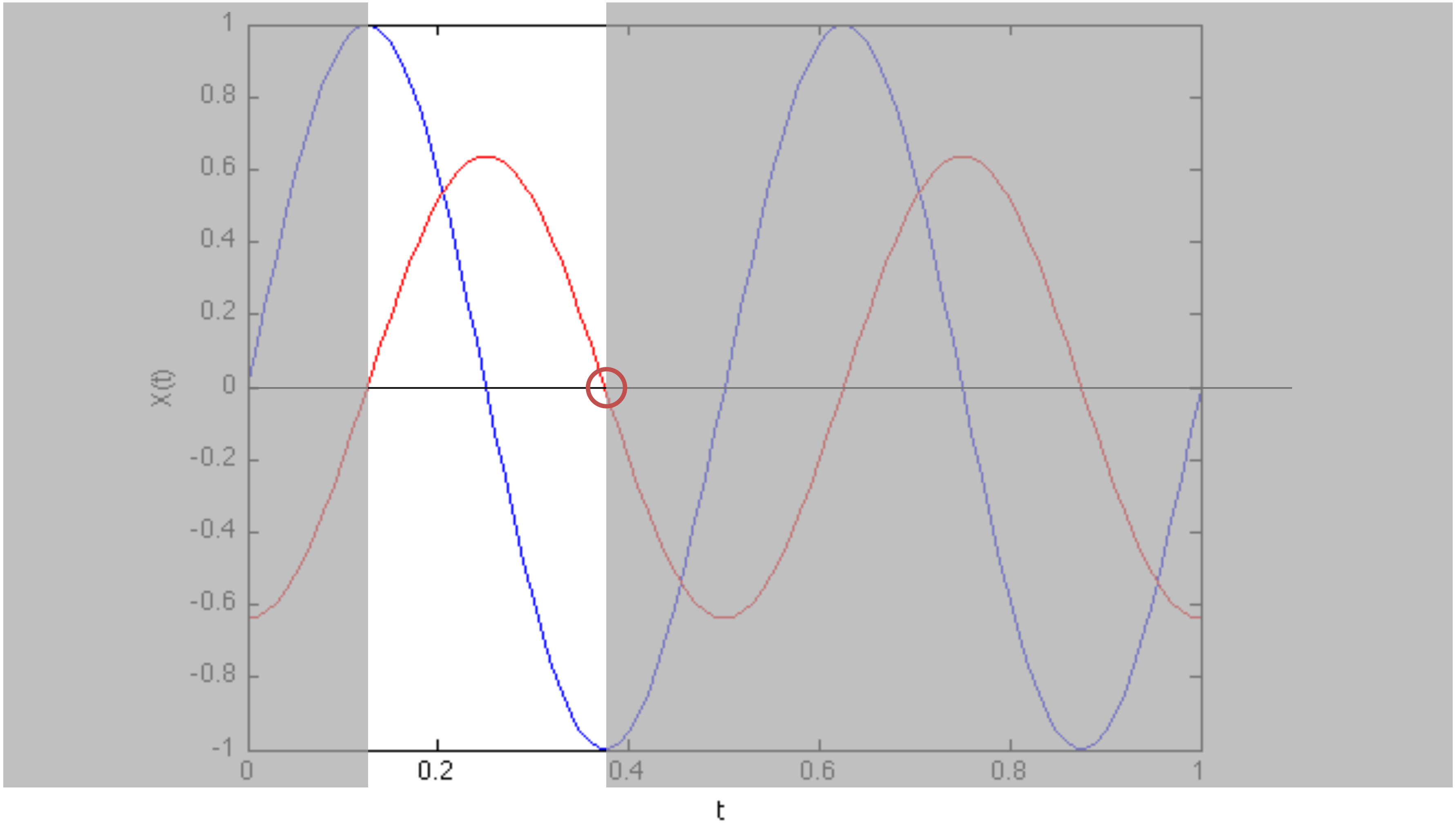
(Here, f = 2 Hz, T=0.5s T_c = 0.25s)

Moving Average Filter (Filtre à moyenne mobile)



$$\hat{X}(t) = \frac{1}{T_c}\int_{t-T_c}^{t} \sin(2\pi f s)ds$$ (Here, f = 2 Hz, T=0.5s T_c = 0.25s)

Moving Average Filter (Filtre à moyenne mobile)



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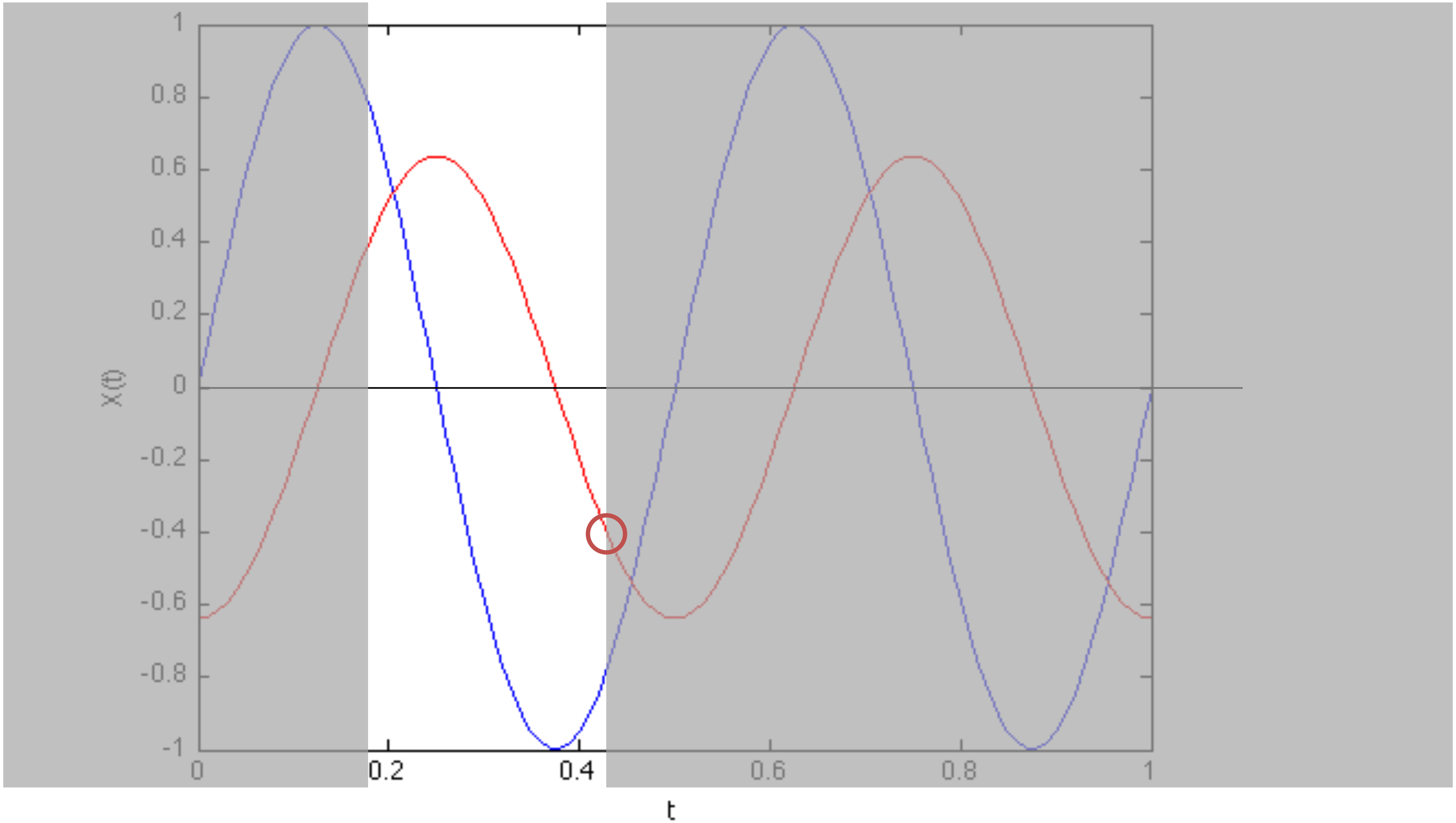
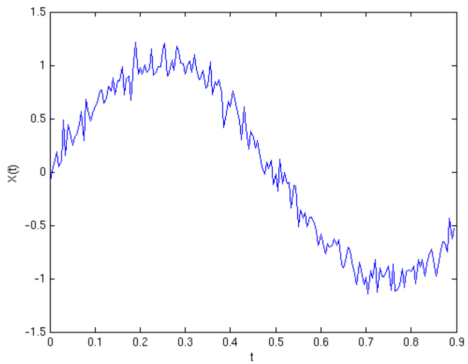
Moving Average Filter (Filtre à moyenne mobile)

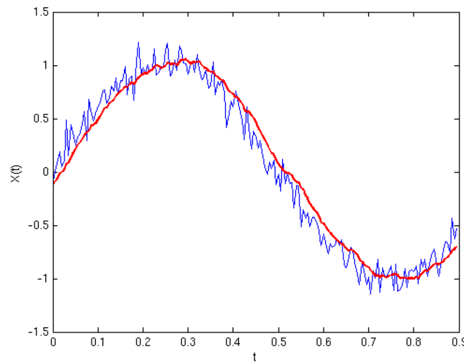


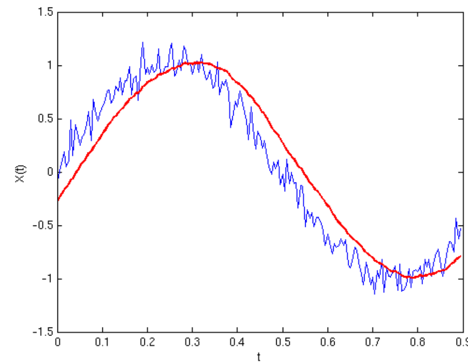
(Here, f = 2 Hz, T=0.5s T_c = 0.25s)

Moving Average Filter

Another example: $X(t) \rightarrow \hat{X}(t)$





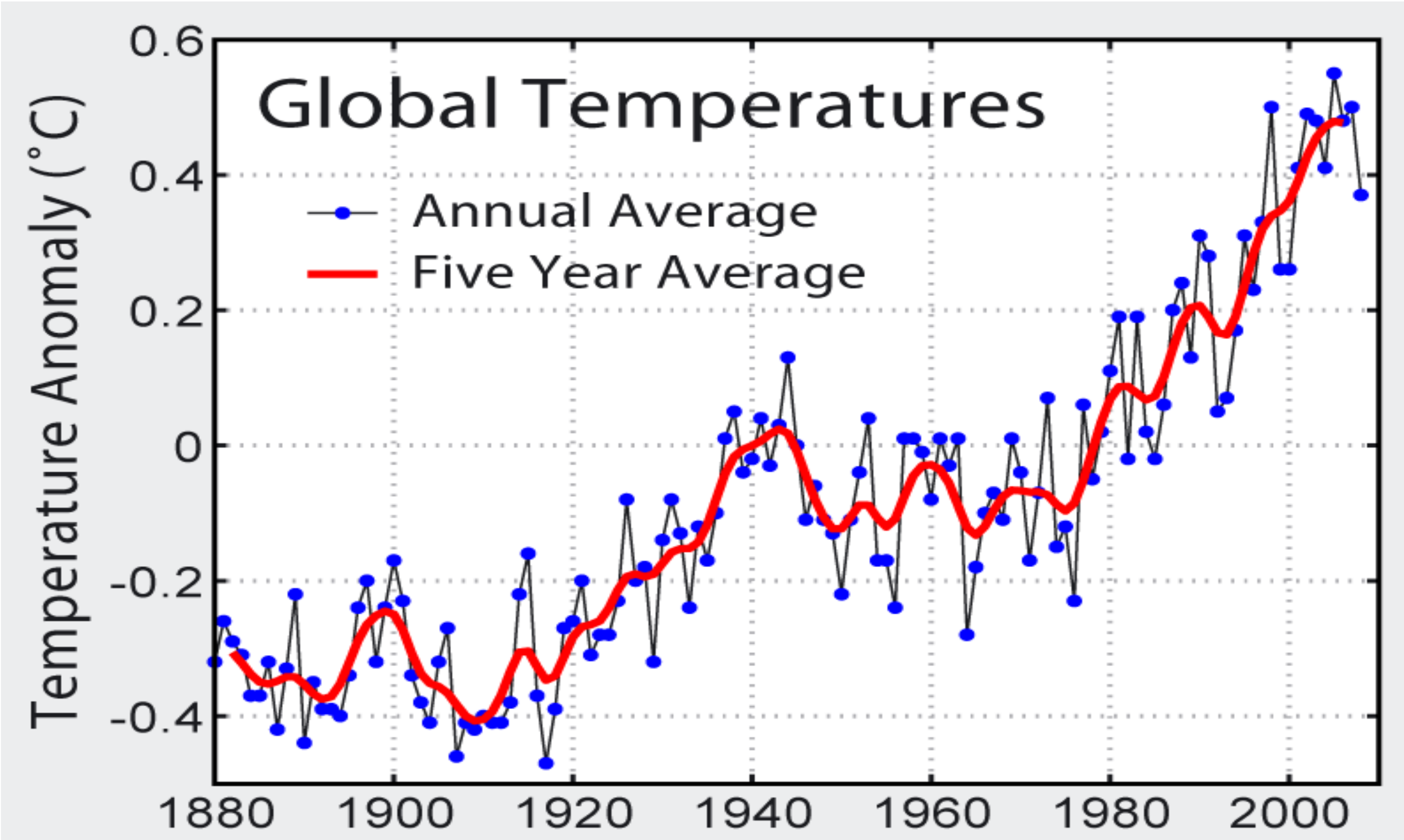


T_c = 0.05sec T_c = 0.1sec

The higher T_c is, the more regular (smoother) the output signal is but also the higher the delay is.

Moving Average Filter

Another example:



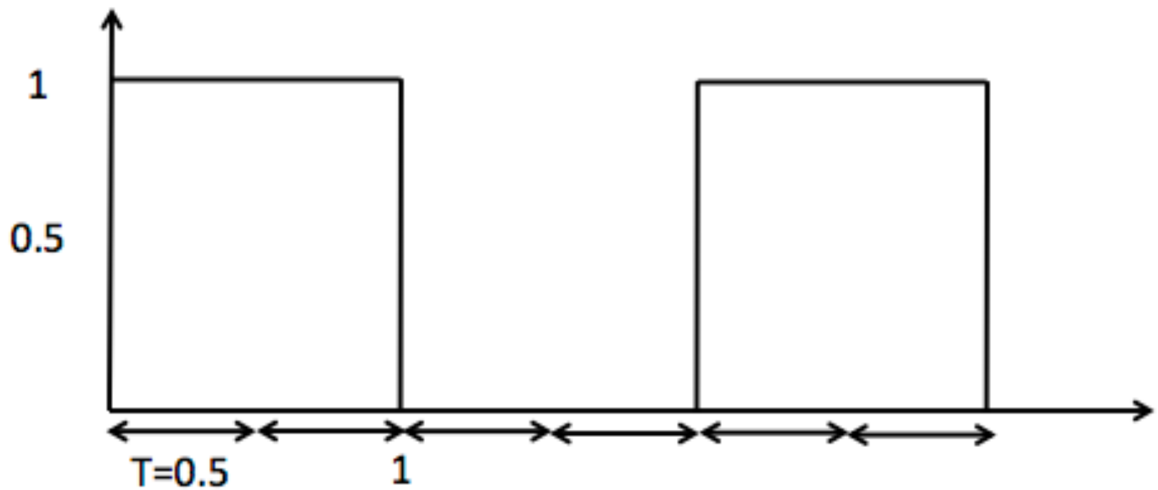
source: Global Warming Art

Global average surface temperature 1880 to 2009,
with zero point set at the average temperature between 1961 and 1990.

Example

$$\hat{X}(t) = \frac{1}{T_c}\int_{t-T_c}^{t} X(s)ds$$

- Consider the following signal $X(t)$ with period 4*T. Into what signal is X(t) transformed if it passed through a moving average filter with T_c = T ?



Example

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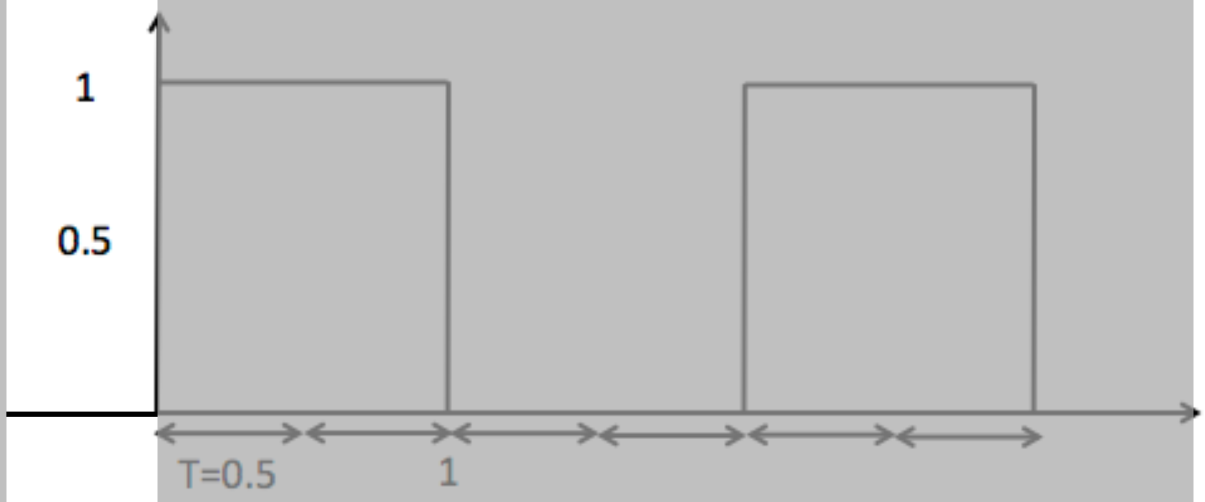
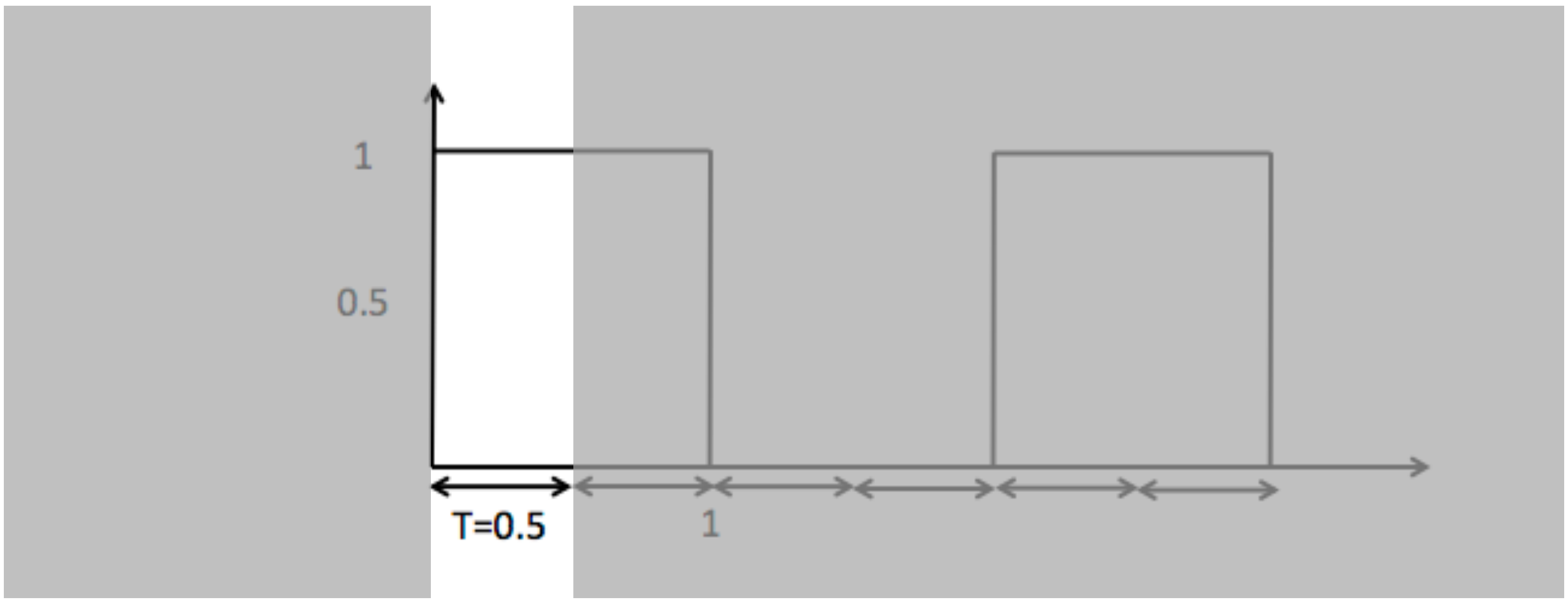


$$\hat{X}(0) = 0$$

Example

$$\hat{X}(t) = \frac{1}{T_c}\int_{t-T_c}^{t} X(s)ds$$

- Consider the following signal $X(t)$ with period 4*T. Into what signal is X(t) transformed if it passed through a moving average filter with T_c = T ?

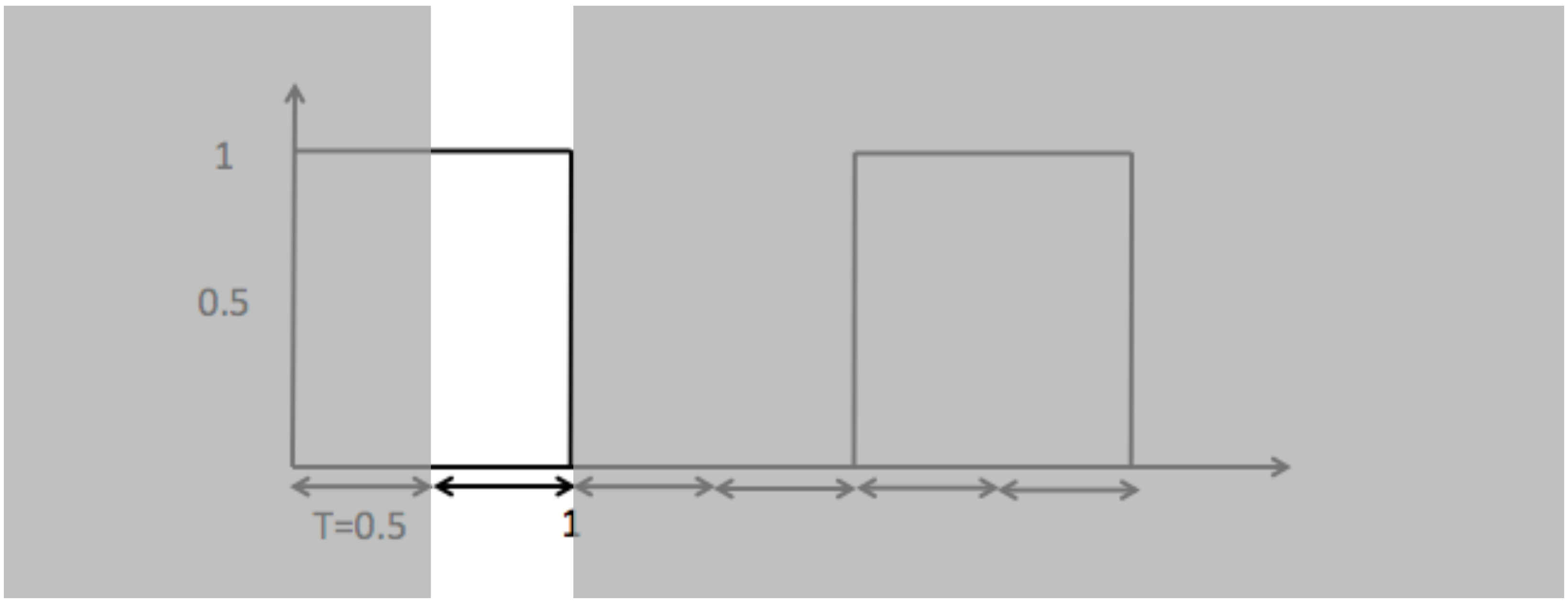


$$\hat{X}(0.5) = \frac{1}{T}\cdot T \cdot 1 = 1$$

Example

$$\hat{X}(t) = \frac{1}{T_c}\int_{t-T_c}^{t} X(s)ds$$

- Consider the following signal $X(t)$ with period 4*T. Into what signal is X(t) transformed if it passed through a moving average filter with T_c = T ?

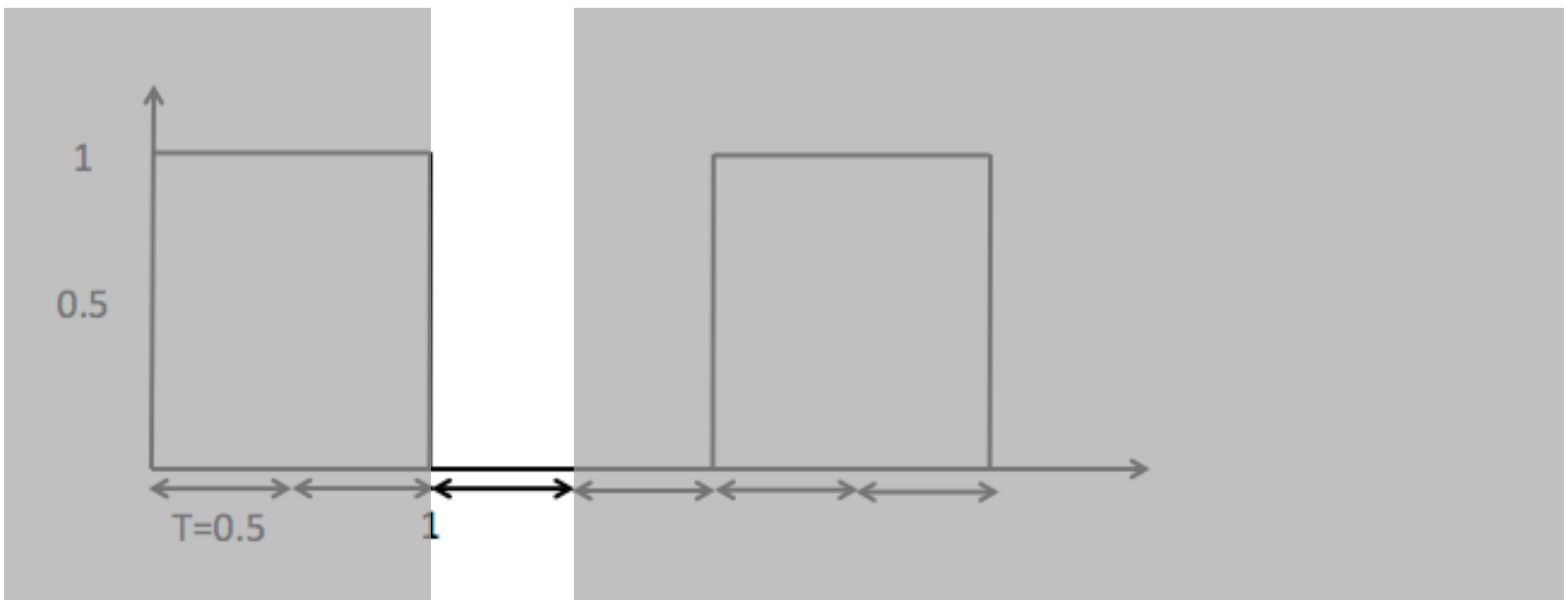


$$\hat{X}(1) = \frac{1}{T} \cdot T \cdot 1 = 1$$

Example

$$\hat{X}(t) = \frac{1}{T_c}\int_{t-T_c}^{t} X(s)ds$$

- Consider the following signal $X(t)$ with period 4*T. Into what signal is X(t) transformed if it passed through a moving average filter with T_c = T ?

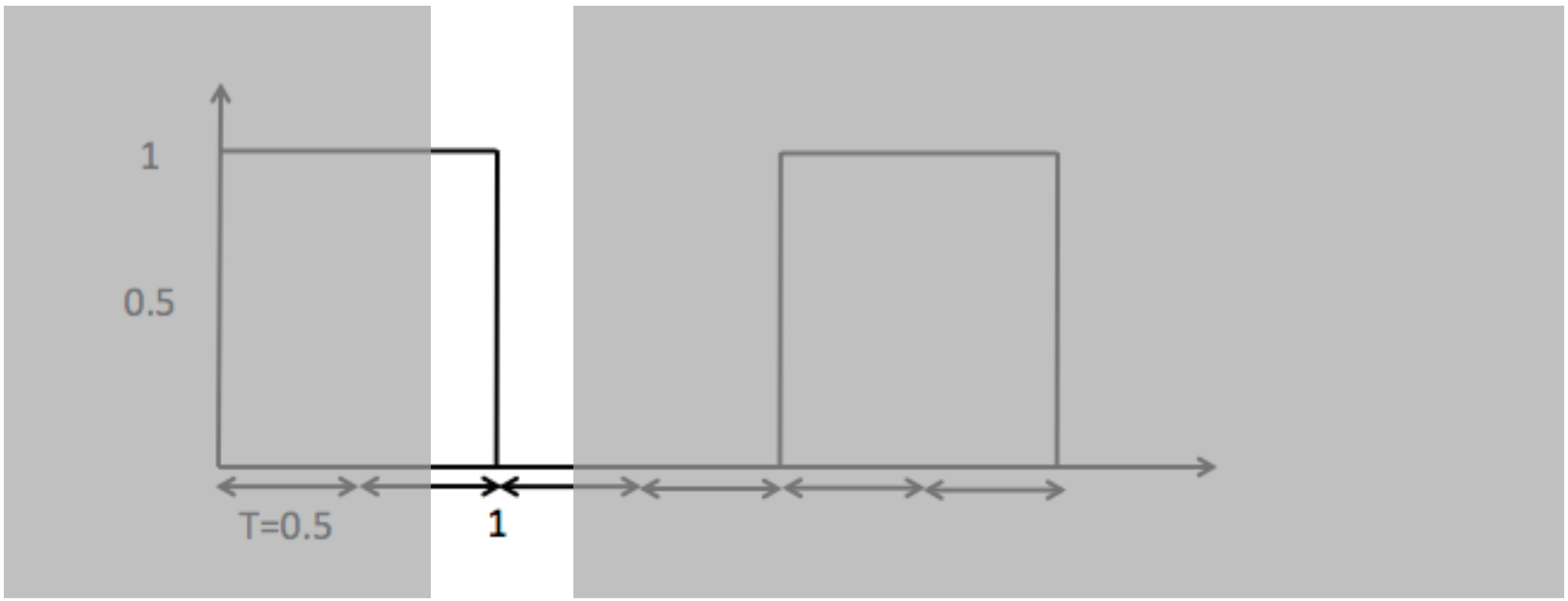


$$\hat{X}(1.5) = 0$$

Example

$$\hat{X}(t) = \frac{1}{T_c}\int_{t-T_c}^{t} X(s)ds$$

- Consider the following signal $X(t)$ with period 4*T. Into what signal is X(t) transformed if it passed through a moving average filter with T_c = T ?

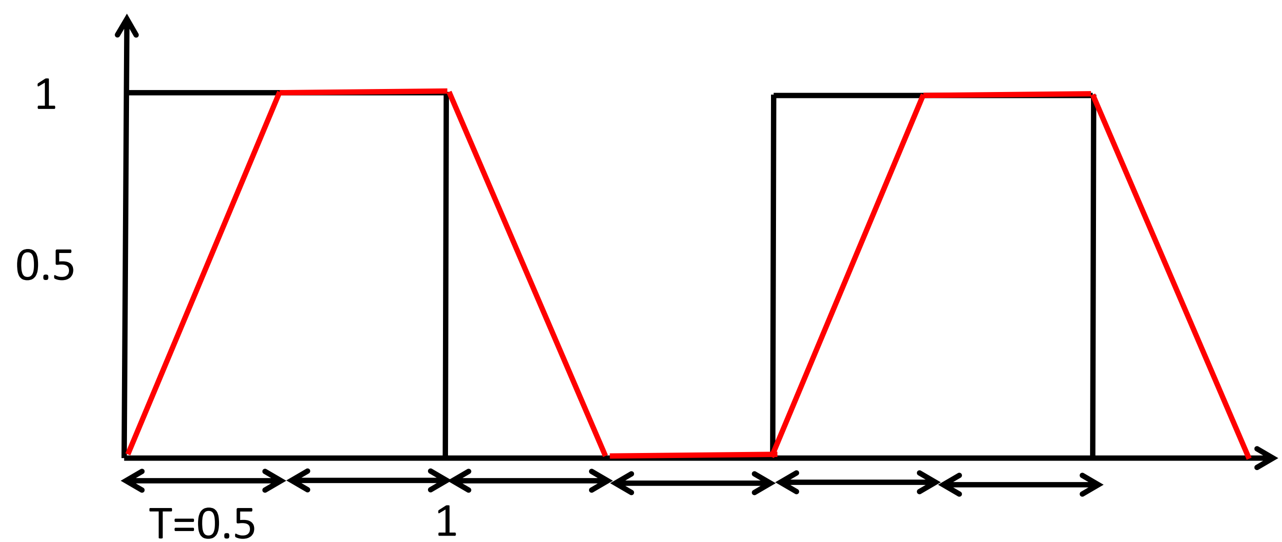


$$\hat{X}(1) = \frac{T}{2}\cdot\frac{1}{T} = \frac{1}{2}$$

Example

$$\hat{X}(t) = \frac{1}{T_c}\int_{t-T_c}^{t} X(s)ds$$

- Consider the following signal $X(t)$ with period 4*T. Into what signal is X(t) transformed if it passed through a moving average filter with T_c = T ?



Integral of a constant c is a line.

Moving Average Filter

Let us consider again a sinusoid:

Recall: $\int \sin(a \cdot x)dx = -\frac{1}{a}\cos(a \cdot x)$

$cos(b) - cos(a) = 2\ sin((a\text{-}b)/2)sin((a+b)/2)$

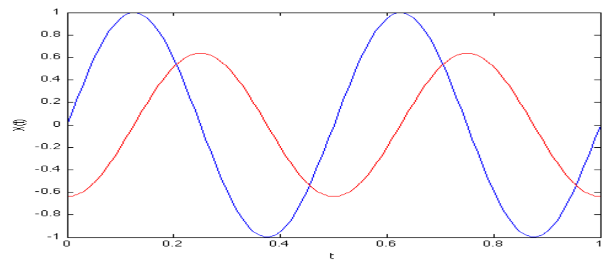
$X(t) = \sin(2\pi f\,t)$

$$\hat{X}(t) = \frac{1}{T_c}\int_{t-T_c}^{t} \sin(2\pi fs)ds$$

$$= \frac{\cos(2\pi f(t-T_c)) - \cos(2\pi ft)}{2\pi fT_c}$$

$$= \frac{\sin(\pi fT_c)}{\pi fT_c}\sin(2\pi ft - \pi fT_c)$$

It follows that $\forall t \in \mathbb{R}, \max\left|\hat{X}(t)\right| \leq \frac{1}{\pi fT_c}$



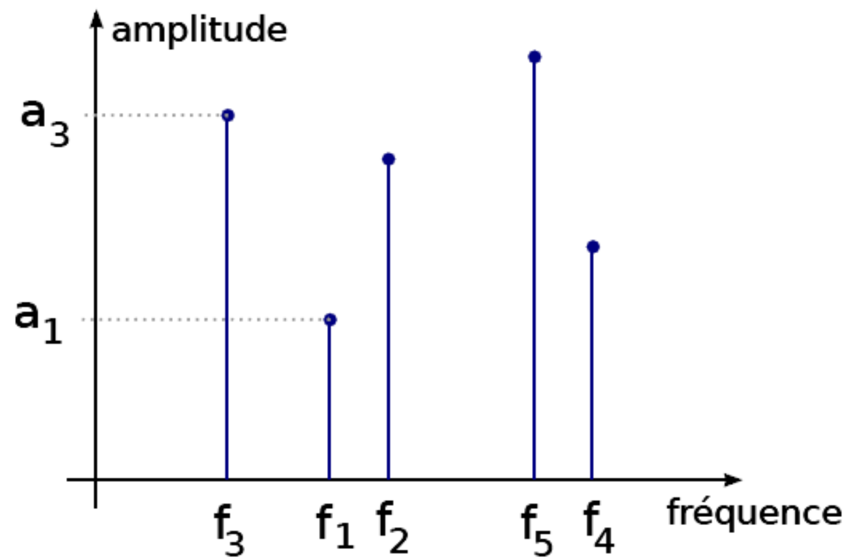
f = 2 Hz, T_c = 0.25 s

A = sin($\pi \cdot 2 \cdot 0.25$) / ($\pi \cdot 2 \cdot 0.25$) = sin(π / 2) / (π / 2) = 2/π

We can see that if $f\ T_c$ is large, the resulting amplitude is small. In particular, if f is large, then $f\ T_c$ is large and therefore high frequencies are filtered (silenced).

Recall: Frequency Spectrum

- In the “frequency space”:
  - Horizontal axis = frequency
  - Vertical axis = amplitude

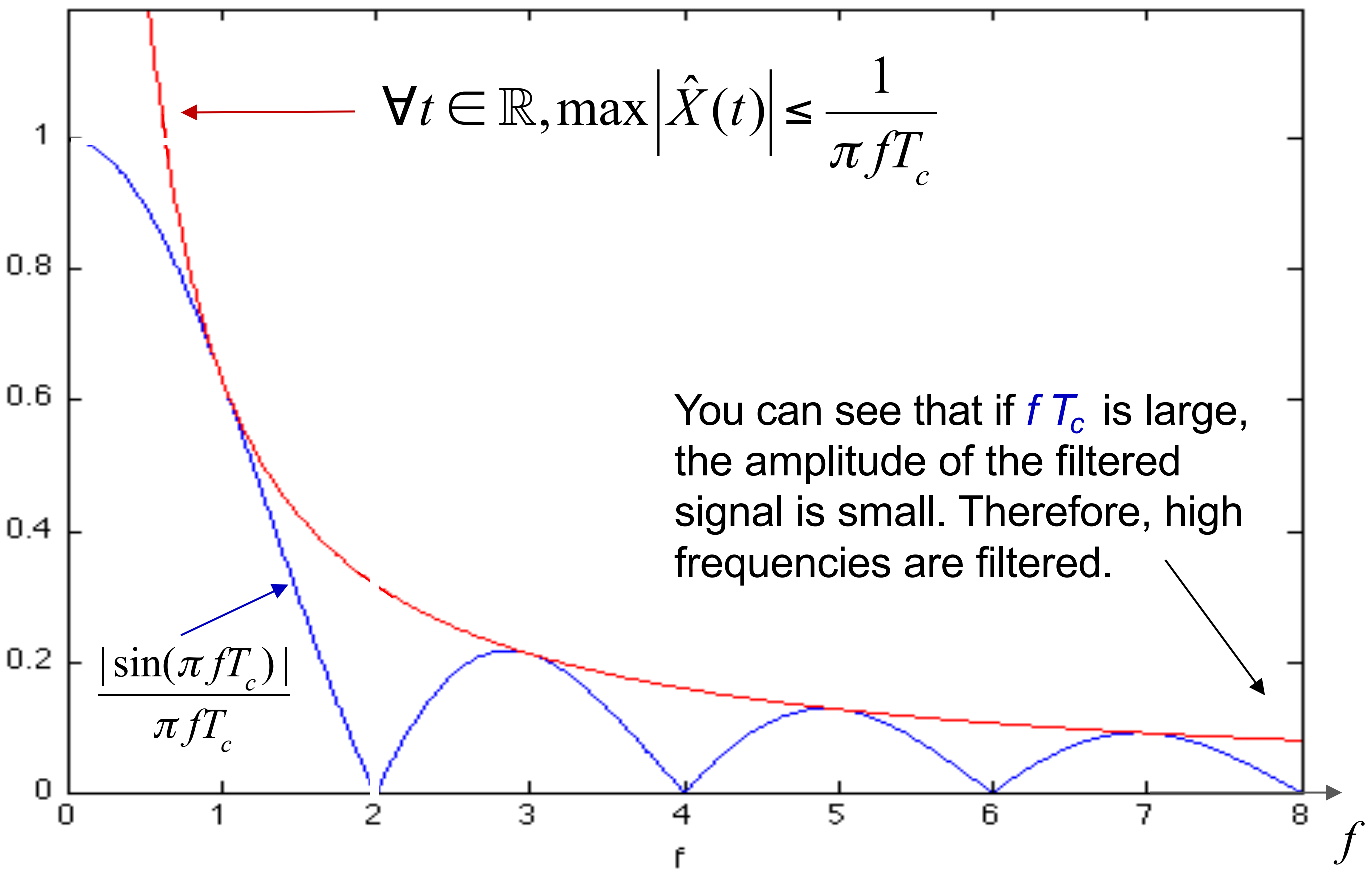


- Example: a sum of sinusoid:
  $X(t) = a_1 \sin(2\pi f_1 t + \delta_1) + \ldots + a_n \sin(2\pi f_n t + \delta_n)$

- This representation is called the **spectrum** of the signal

Moving Average Filter

We have concluded that the maximum amplitude of the filtered signals is bounded by the function $1/\pi f T_c$ shown in red below for $T_c = 0.5$s (meaning f_c=2Hz)



You can see that if $f\, T_c$ is large, the amplitude of the filtered signal is small. Therefore, high frequencies are filtered.

A special case: if T_c ist a multiple of the period $T = 1/f$, the value of the integral is zero because the average of a sinus signal over one (or multiple) period(s) is zero.

We can also see in the blue curve that $\sin(\pi f T_c)$ is approaching 0, if $\pi f T_c = (0.5 f \pi)$ is approaching $K\pi$ for any integer K.

Comparison

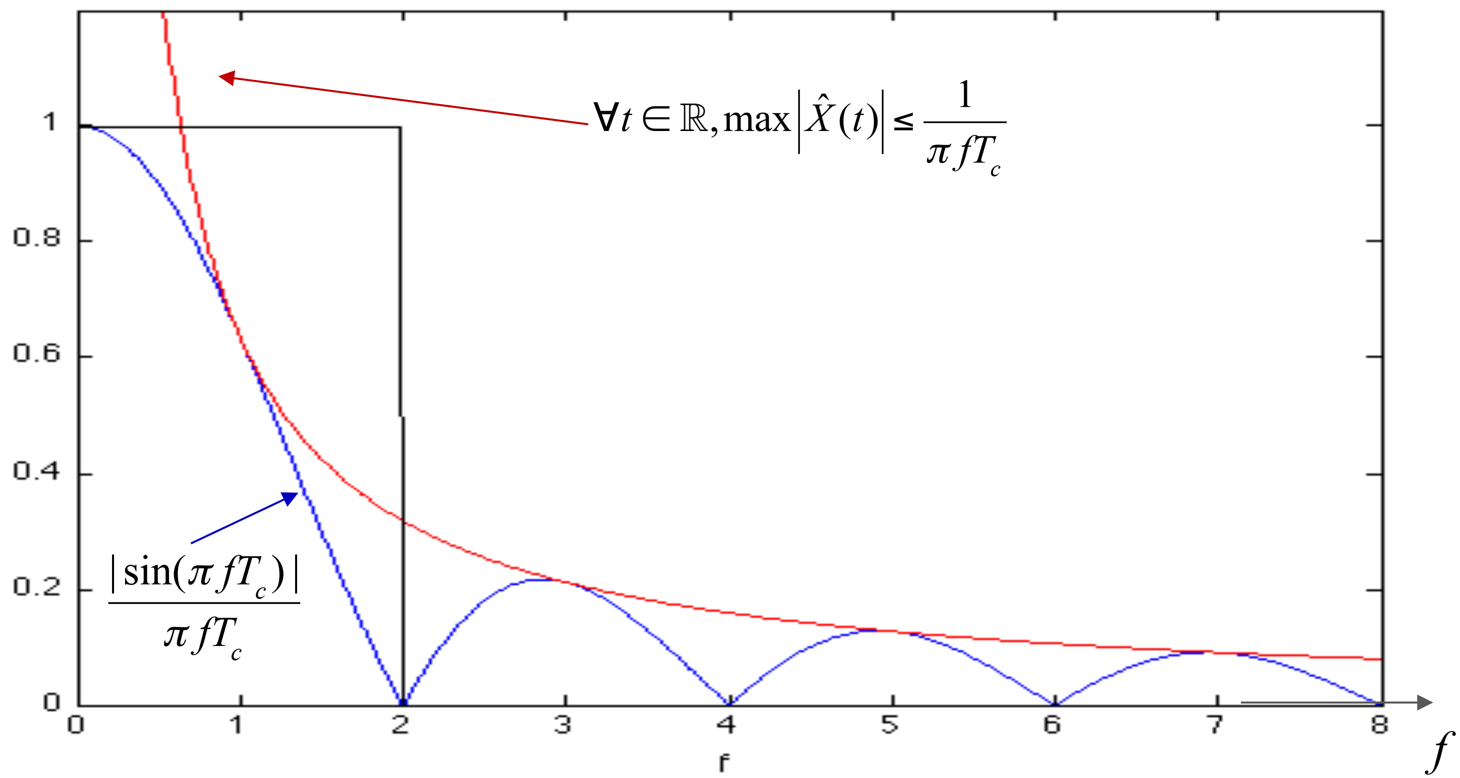
Let’s compare the frequency attenuation of

- an ideal low pass filter with a cutoff frequency of f_c = 2 Hz and
- an moving average filter with an integration period $T_c = 1/f_c = 0.5$ s

$$X(t) = \sin(2\pi f t)$$

$$\hat{X}(t) = \frac{1}{T_c}\int_{t-T_c}^{t} \sin(2\pi f s)ds$$

$$= \boxed{\frac{\sin(\pi f T_c)}{\pi f T_c}} \sin(2\pi f t \boxed{- \pi f T_c})$$



Filters: Conclusion

- Low pass filters are used to suppress or attenuate high frequency in a signal
- Ideal low pass filters cannot be constructed
- A moving average filter is a low pass filter.
- We will soon see an important application of low pass filters

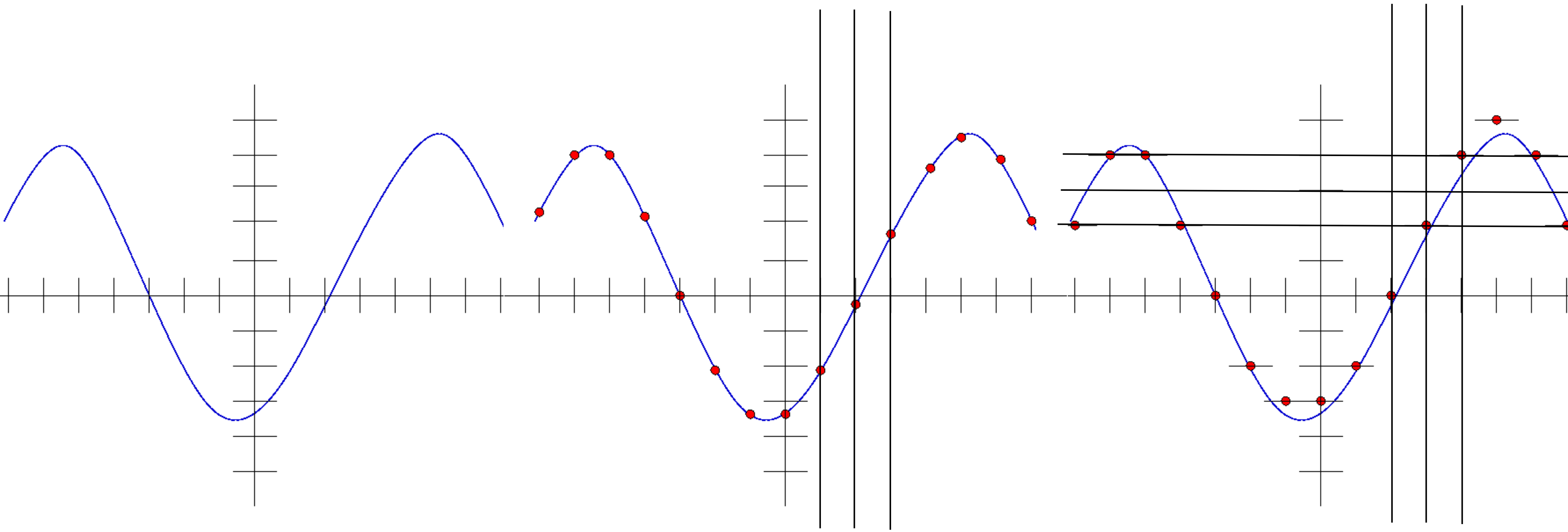
Sampling (Echantillonnage)

Sampling (Echantillonnage)

- We return now to our initial question:
  - How to represent or capture a physical reality with bits?
- All signals that surround us are of analog nature (e.g., sound, electromagnetic waves, movement of engines…)

- A computer can work only with digital data.
- In order to allow a computer to process (e.g., analyze, modify, store…) a signal $(X(t), t \in \mathrm{R})$ we have to
  1. sample the signal at discrete time instances
  2. quantify the value of the signal at these instances
- A natural question to ask is: what will we lose if we sample and quantify a signal?

Sampling and Quantifying



Original signal

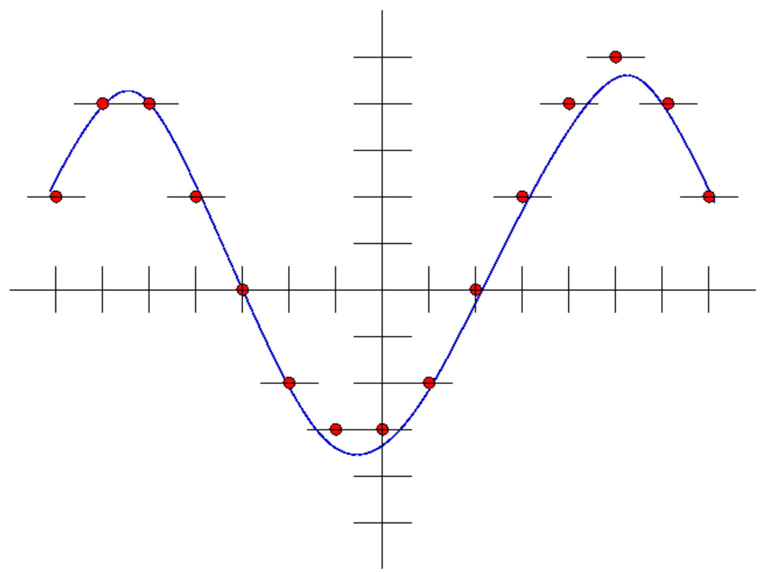
Sampled signal

Sampled and quantified signal

Recall: floating point numbers are used to represent real values, e.g., 3.4
However, we cannot represent all real numbers correctly, e.g., we might represent 3.46788 by 3.46. The same approximation happens if we quantify a signal value.

Sampled and Quantified Signals

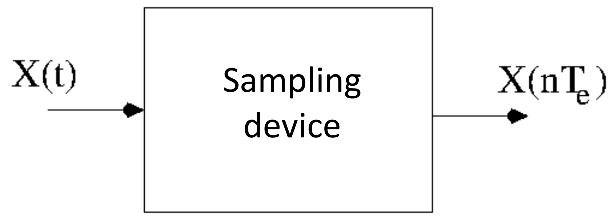
- A sampled and quantified signal is a list of value: $x_0, x_1, x_2, \ldots$
- One value per sampling point
- Each value can be represented using a fixed number of bit (cf. lecture on "Information Representation")



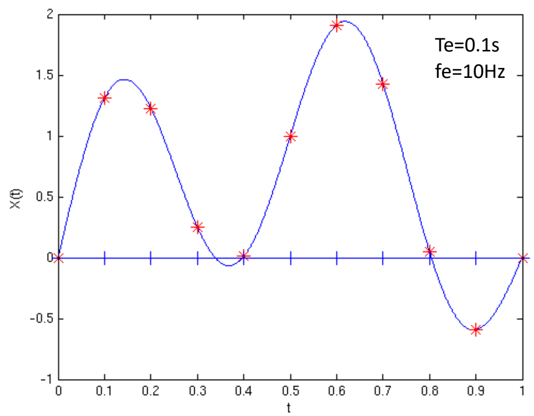
Assume we sample a signal for 10 s with 10 kHz and store each sampling point with 32 bits (4B). What is the size of the sampled signal? $10 \cdot 10'000 \cdot 4 = 400'000$ B = 400 kB

Sampling

- In the following, we focus on the sampling a signal



Input signal $(X(t), t \in \mathrm{R})$ $\rightarrow$ sampled signal $(X(nT_e), n \in \mathrm{Z})$:



T_e = sampling period, $f_e = 1/T_e$ = sampling frequency

Sampling Period Te

- What is the right sampling period T_e?

- If T_e is too small: too much information to process
- If T_e is too large: information is lost

What happens if Te is too large?

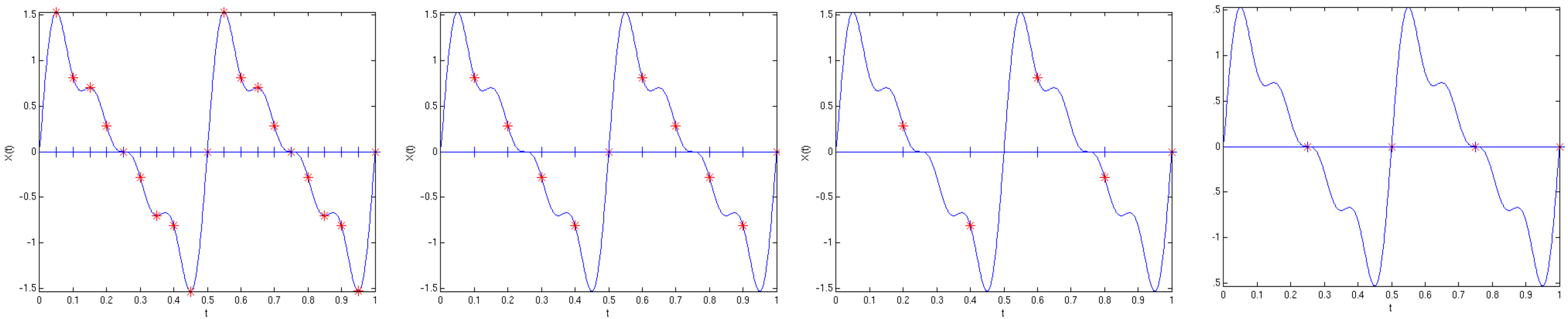
- Example: let's consider the signal that we saw before

$$X(t) = \sin(4\pi t) + \frac{1}{2}\sin(8\pi t) + \frac{1}{3}\sin(12\pi t) + \frac{1}{4}\sin(16\pi t)$$

What happens if Te is too large?

- Example: let's consider the signal that we saw before

$$X(t) = \sin(4\pi t) + \frac{1}{2}\sin(8\pi t) + \frac{1}{3}\sin(12\pi t) + \frac{1}{4}\sin(16\pi t)$$

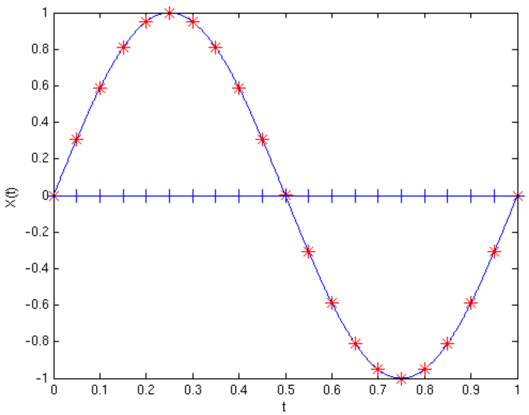


$T_e = 0.05$ s $\quad$ $T_e = 0.1$ s $\quad$ $T_e = 0.2$ s $\quad$ $T_e = 0.25$ s

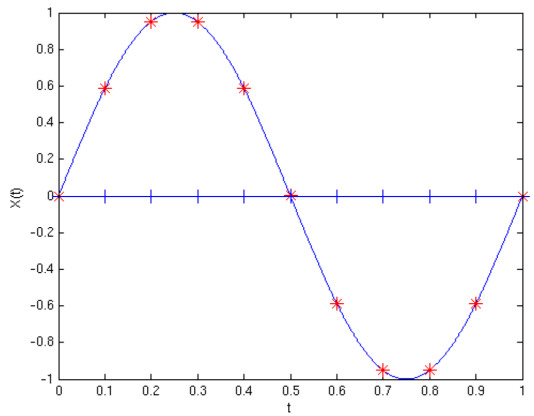
Sampling period *Te*

Sampling a Sinusoid

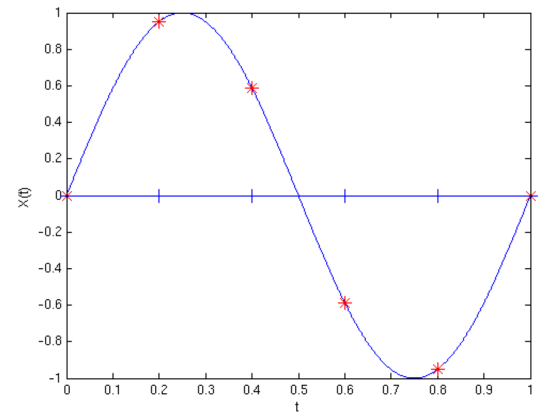
Another example: sinusoid $X(t) = \sin(2\pi t)$ $(f = 1\,\text{Hz})$



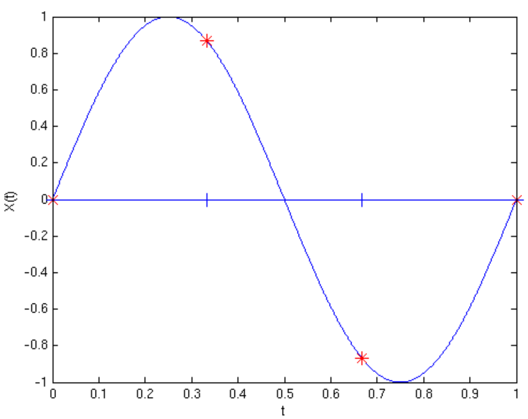
$T_e = 0.05$ s



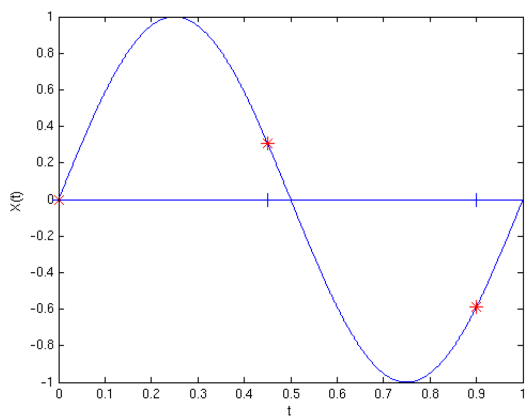
$T_e = 0.1$ s



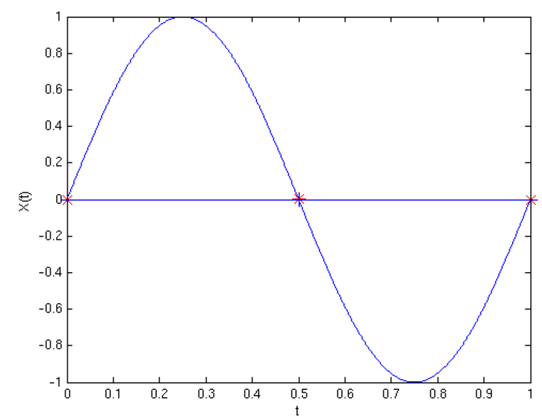
$T_e = 0.2$ s



$T_e = 0.33$ s



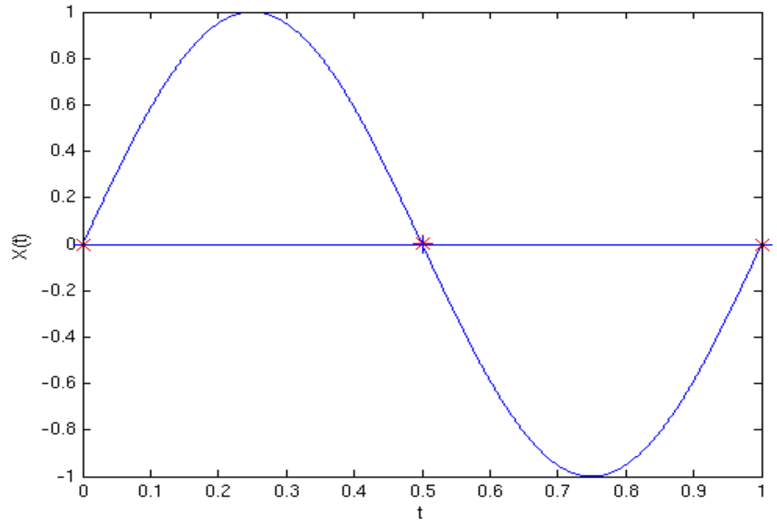
$T_e = 0.45$ s



$T_e = 0.5$ s

Sampling a Sinusoid

Another example: sinusoid $X(t) = \sin(2\pi t)$ $(f = 1\,\text{Hz})$



- Sampling period $T_e = 0.5\ s$
- In order to be able to reconstruct this sinusoid from a sampled signal, T_e has to be smaller than 0.5 s, which means the sampling frequency $f_e = 1/T_e$ has to be larger than 2 (i.e., 2 * frequency)

Sampling a Sinusoid

- In general, the following is true:

  Given a sinusoid *X(t)* with frequency *f* and a sampled version of this signal that was sampled with frequency f_e, then the condition

  $$f_e > 2f$$

  has to be true in order to be able to reconstruct the signal.

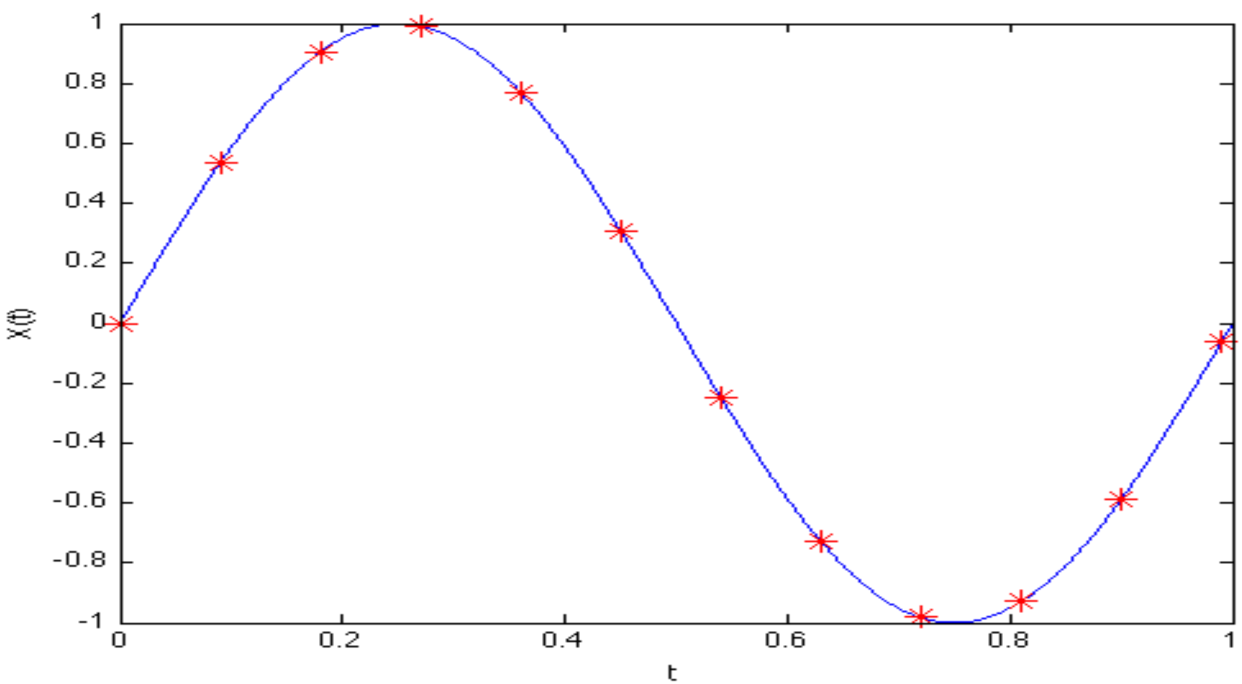
- The **Sampling Theorem** that we will see in the next part, says that this condition is not only **necessary** but also **sufficient**.
- We will also see that this theorem applies to all signals not just sinusoids.

Application

- On a CD/DVD, the sound is sampled with a frequency of 44.1 kHz because the human ear can (in general) not hear frequencies above 20kHz.

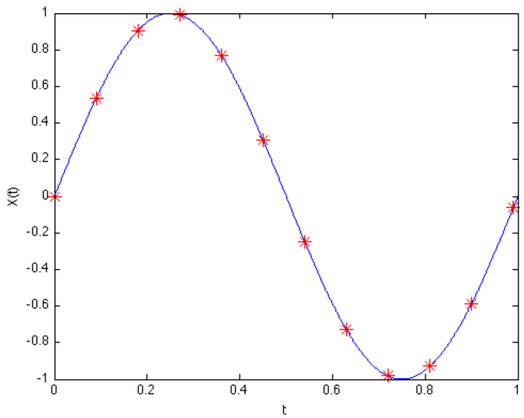
What happens if fe < 2f ?

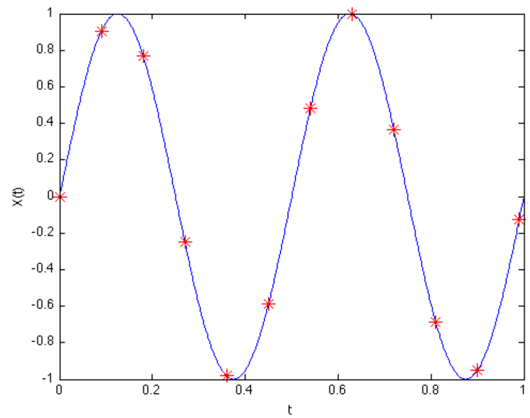
- What happens if the sampling frequency is too low? When the signal is **sub-sampled.**
- Let's reconsider our example with a sinusoid with 1Hz: $X(t) = \sin(2\pi t)$ and a sampling period of $T_e = 0.09\ s$, i.e., a sampling frequency of $f_e = 1/T_e = 1/0.09 = 100/9 \approx 11.11Hz$

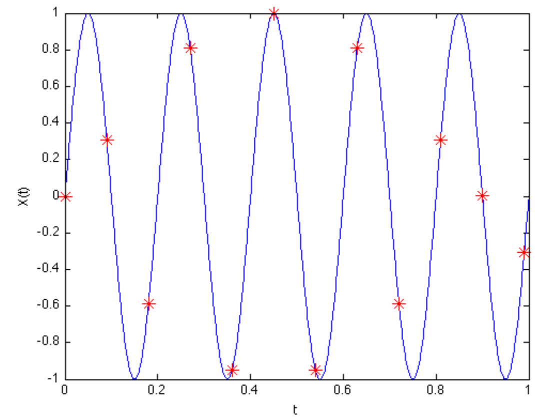


What happens if fe < 2f ?

$f_e \approx 11.11Hz$







f = 1 Hz

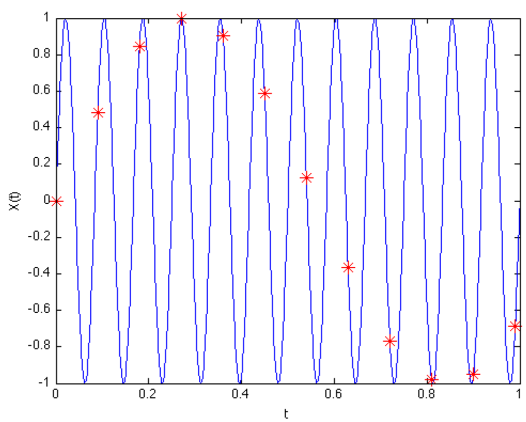
f = 2 Hz

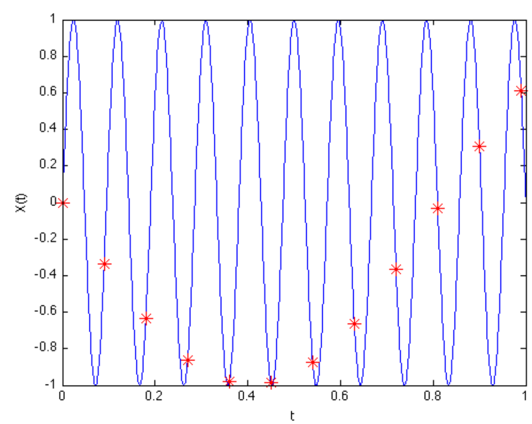
f = 5 Hz

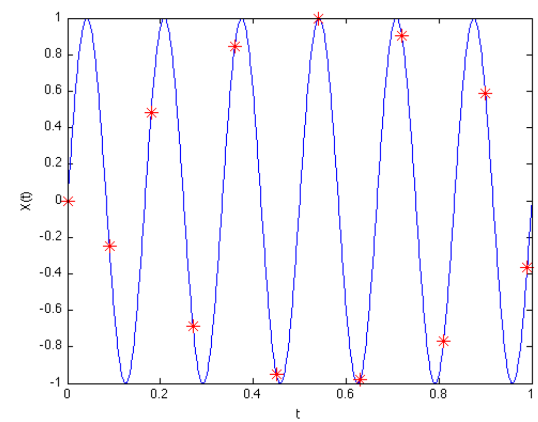
f = 12 Hz

f = 10.5 Hz

f = 6 Hz

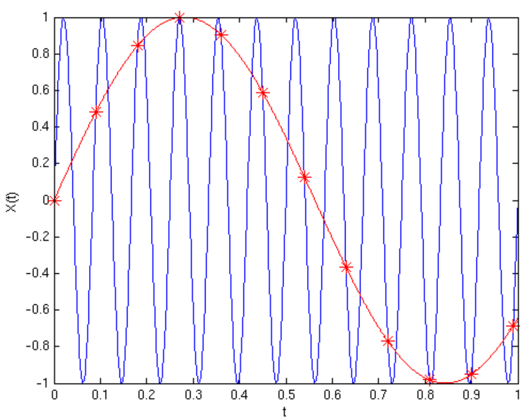


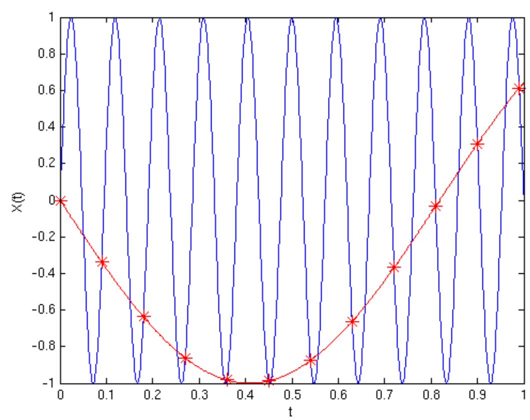




What happens if fe < 2f ?

- In the two last cases, we saw other signals appearing, e.g.,
  - a sinusoid with a **lower** frequency
  - a sinusoid with a **lower** frequency that initially **decreases**.





https://www.youtube.com/watch?v=r3hs8pPCQmo

http://www.youtube.com/watch?v=jHS9JGkEOmA

- This phenomena is called **stroboscopic effect (or aliasing)** and it appear if we sub-sample a signal. We will discuss it in detail in the next part.

What happens if fe < 2f ?

Example of a patter on a wall

Another example with tissue http://www.youtube.com/watch?v=jXEgnRWRJfg

Summary

- Signals and Sinusoids (sine waves)
- All interesting signals are sums of sinusoids
- Frequencies present in a signal (bandwidth and spectrum)
- Filters and sampling
- Necessary condition to reconstruct: $f_e > 2f$

- Next:
  - How to reconstruct a signal from a sampled version?
  - Sampling Theorem
  - Sub-Sampling