Week 4: Sampling of signals II (Solutions)

1 Filters

1. FALSE - Frequencies between f_1 and f_2 will be pass through.
2. TRUE - All frequencies below f_1 and above f_2 (consequently also above and including f_1, since $f_2 < f_1$) will be attenuated.
3. TRUE - All frequencies below and including the highest frequency will be attenuated by the high-pass filter.
4. FALSE - The highest frequency will not be attenuated by the low-pass filter, since it is smaller than f_2. If highest frequency is bigger than f_1 and $f_1 < f_2$, it will also not be attenuated by the high-pass filter.

2 Interpolation Formula

1. TRUE - According to the sampling theorem.
2. FALSE - This describes linear interpolation. Given the interpolation formula $X_I(t) = \sum_{m\in\mathbb{Z}} X(mT_e)\mathrm{F}(\frac{t-mT_e}{T_e})$, this is achieved by F being the following function (instead of $F(t) = sinc(t)$):

$$F(t) = \begin{cases} 1 - |t|, & \text{if } |t| \leq 1 \\ 0, & \text{if } |t| \geq 1 \end{cases}$$

3. TRUE - Since $F(0) = 1$ and $F(k) = 0$ for all $k \in Z|k \neq 0$
4. TRUE - According to the sampling theorem.

3 Stroboscopic Effect

1. FALSE - We need to avoid the stroboscopic effect.
2. TRUE - To avoid the stroboscopic effect, where frequencies higher than $(f_e/2)$ would be "folded back".
3. FALSE - Frequencies below $f_e/2$ can be reconstructed.
4. FALSE - This is costly and impractical if frequencies go up to infinity.

4 Signals

1. FALSE - Amplitude is the same
2. FALSE - Amplitude is the same
3. TRUE - The period of X2 (0.5s) is half of the period of X1 (1s). Thus, the frequency of X2 is double the frequency of X1.
4. FALSE - Halving the frequency would result in doubling the period. We want to achieve the opposite.

5 Sampling and Signal Reconstruction

1. The samples of s_0 are:
   $\hat{s}_0(0) = \sin(0) = 0$
   $\hat{s}_0(1) = \sin(\frac{2\pi 440}{1320}) = \sin(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}$
   $\hat{s}_0(2) = \sin(\frac{2\pi 440 \cdot 2}{1320}) = \sin(\frac{4\pi}{3}) = -\frac{\sqrt{3}}{2}$
   $\hat{s}_0(3) = \sin(\frac{2\pi 440 \cdot 3}{1320}) = \sin(2\pi) = 0$
   The sampling frequency is a multiple of the signal frequency, hence the sampled values are periodic.
   The samples of s_1 are:
   $\hat{s}_1(0) = \sin(0) = 0$
   $\hat{s}_1(1) = \sin(\frac{2\pi 880}{1320}) = \sin(\frac{4\pi}{3}) = -\frac{\sqrt{3}}{2}$
   $\hat{s}_1(2) = \sin(\frac{2\pi 880 \cdot 2}{1320}) = \sin(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}$
   $\hat{s}_1(3) = \sin(\frac{2\pi 880 \cdot 3}{1320}) = \sin(4\pi) = 0$
   Again, the samples are periodic.

2. For s_0, from the sampling theorem we know that the interpolation gives back the original signal, because $1320Hz > 2 \cdot 440Hz$. For s_1, under-sampling occurs. We see that $\hat{s}_1(k) = -\hat{s}_0(k)$, thus the reconstructed signal is $-s_0(t) = \sin(2\pi 440t + \pi)$.

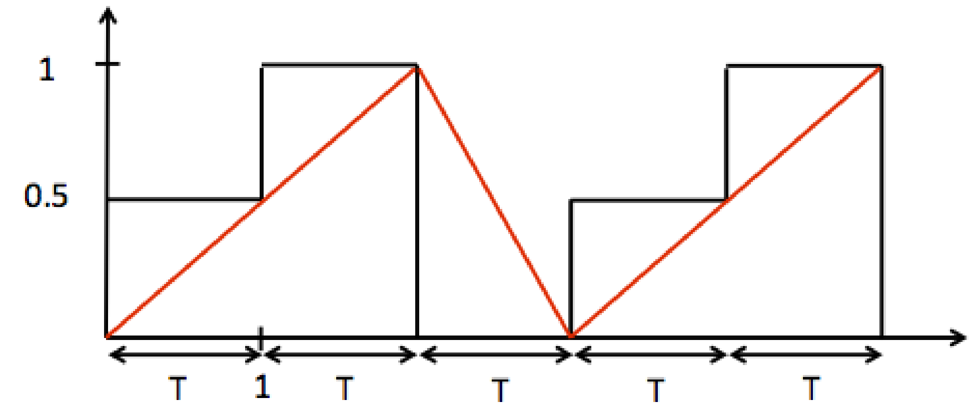
3. The samples of s_2 are $\hat{s}_2(k) = \hat{s}_0(k) + \hat{s}_1(k) = 0$. The reconstructed signal is a constant 0.

6 Moving Average Filter

Recall a moving average filter computes the average of the given function over the last T_c time units. The average is computed by "summing all the values" in the interval $[t - T_c, T]$ and dividing them by T_c. We use a definite integral (i.e., an integral with borders) to "sum all values" of an analog signal.

In the first example, the period of the moving average filter is 1, i.e, $T_c = 1$, which means that to get the value at time t of the filtered function X_f, we have to compute the integral of the input function from t-1 to t and divide the result by 1. Recall that a definite integral computes the area below the function. So, let us compute a few points of our filtered function X_f for $t = 0, t = 1, t = 2$, and $t = 2.5$. At $t = 0$, we need to compute the area below the function from -1 to 0. Since the function is periodic, its values in the interval [-1,0] are the same as in the interval [2,3], meaning 0. So, $X_f(0) = 0$. The value of $X_f(1)$ is the area below the function in the interval 0 to 1. This area is a rectangle with a width of 1 and height of 0.5, so the area is 0.5 and therefore $X_f(1) = 0.5$. Between $t = 0$ and $t = 1$, X_f is constantly increasing because X is constant. The value of $X_f(2)$ is the area below the function in the interval 1 to 2; this is a rectangle with a width of 1 and height of 1, so $X_f(2) = 1$. Finally, $X_f(2.5)$ is the area below the function in the interval 1.5 to 2.5, which is a rectangle with width 0.5 and height 1, so $X_f(2.5) = 0.5$. You can apply the same reasoning for the rest of the function and the second example.

Alternatively, you can reason that, e.g., the value of X_f at $t = 1$ is the average value in the interval $[0, 1]$ and because $X(t) = 0.5$ for all values in this interval, we have that $X_f(t) = 0.5$. You can apply the same reasoning for $X_f(2)$ and $X_f(3)$. Use a combination of these two techniques to draw the red line shown in the figure below.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1
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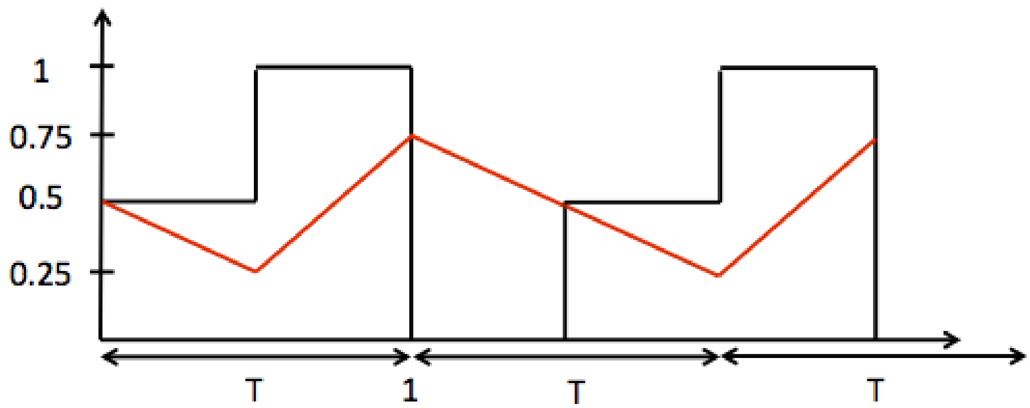
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